# *Tesla S P85D Ludicrous Mode Performance Simulation*

The Simulation can be run or modified with Mathcad 14/15. Free Trial at: **http://www.ptc.com/product/mathcad/free-trial**
**Mathcad Simulation at: http://www.LeapCad.com/TeslaSP85DLudicrousModeSimulation.xmcd** 9-7-2015

## Goal: Simulate Ludicrous Mode Peak Acceleration Performance

This paper shows a macro model for performance simulation of a Tesla S P85D Ludicrous Mode. The key parameters are peak motor torque, peak battery power (SOC), curb weight, maximum tire traction, and some assumptions about the power loss/efficiency: high efficiency induction motors (93%), and Inverter and power train (87%). Net System Efficiency, SysEff ~ **81%**. From statements of Elon Musk, we would infer that tire traction is capable of gripping the road @1.1g. Assume that with sufficient power, we can get a peak Motor Torque of 713 ft lb. For **50% SOC**, battery max power is 724 hp, System **Power of 586 hp**. The model shows that for **50% SOC**, the time from 0 to 60 mph is **2.8 sec**. This Analysis in done in the following ten Sections. Section IV considers four different traction scenarios (Sec IV, pg. 4). Results were calculated for 100, 90, 80, 70, 60, 50% State of Charge (SOC). See last graph pg 5. **NOTE:** The Calculations & graphs in **Sections III to VI** are shown for **100% SOC.**

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## I. Introduction - Simple Analysis

### Examining the Difference Between P85D Insane & Ludicrous Speed Upgrade

| Insane Mode Specifications | Ludicrous Mode Specifications |
|---|---|
| 3.1 seconds 0-60 mph | 2.8 seconds 0-60 mph with Ludicrous Speed Upgrade |
| Peak Acceleration: 1 g | Front and Back Motor Power **Spec**: **762 hp** |
| Front and Back Motor Spec: Power **691 hp** | **Electronic Fuse:** 1500 amp "effective" battery upgrade |
| **Conventional Fuse:** 1300 amp battery limitation | **Battery @50% SOC Peak Power:** 1500 A x 360V = **724 hp** |
| **Battery Peak Power**: 1300 A x 398.6 = **695 hp** | **Peak Motor Torque: 713 ft lb.** |
| **Inverter and/or Algorithm sets max accel ~ 1g.** | **New Inverter Algorithm to maximize Ludicrous acceleration.** |

We assume that the implementation of the **New Algorithm** for Rear Motor Torque & Power Split in Ludicrous Mode, will provide the maximum tire grip acceleration, 1.1g, and not optimize Efficiency, that is, minimize Power: = $TS_{rear}$ = 503/809 = 0.63

What is the minimum acceleration needed to meet the 0 to 60 mph in 2.8 seconds? There are two factors involved in this specification, velocity, v, and time, t. For the sake of this Simple Analysis, let's assume that the acceleration is constant. Now acceleration defined as the rate of change of velocity. In symbols, the constant acceleration, $a._{constant}$ = change of velocity/time. Then the acceleration needed to get to 60 mph in 2.8 seconds, or 60 mph/2.8 s, is a constant 21.43 mph per sec.

Thus a **constant** 21.43 mph/s in sufficient to meet our spec. In units of the earth's gravitation g, 1 g = 21.9 mph/s. We need a constant or "average" of 0.978 g. From Newtons 2nd Law we have that F = m a. Then to accelerate a mass of 5096 lb, we need a Force of 4984 lb or 14" tire radius and 9.73 Gear Ratio, a Motor Torque of 598 ft lbf. Then 713 ft lbf should be more than sufficient.

However, torque or acceleration is not constant. The 713 ft lb is peak torque. Torque falls off with vehicle speed or if the battery power is not sufficient to supply enough power to keep it at its peak. Also, Ludicrous Mode uses a New Algorithm that provides complex Traction Control for max acceleration. Thus a more in-depth analysis is required. Among other things, it requires working out what the torque falloff is with speed. We will find that with a net combined Inverter and Motor Power Efficiency of 81% and high performance 1.1 g tires (implied by Elon Musk), we can meet the Ludicrous Specs.
This more in-depth analysis is what follows.

# II. Macro Performance Model Discussion & Description of the Model

**Macro Model:** Macro Models requires only limited knowledge of internal parameters. We treat the system as a Black Box. That is, we don't know the details of what's inside, just a few fundamental parameters. We are only interested in overall performance. Ignore the intricacies. Simple, but not too simple. May not know what is inside, but regardless, the laws of Physics still apply. We just need basic physical parameters such as:

Vehicle mass (Mcurb), Coefficient of tire friction $\mu$, and radius, Gear Ratio GR, max motor Torque & Power, battery power, and System Power Efficiency (Inverter, Gears, and Motors).
The vehicle also has rotational energy from rotating tires, motor rotor, and gear box.
A factor km, which multiplies the mass, accounts for this added rotational mass.
$M_{curb}$ = 4936 lbm, $\mu$ = 1.1 (equivalently, max g = 1.1), 265/35ZR21 tires, tire radius=14 inches, GR = 9.73.
Then acceleration (a) is given by:

Newton's Second Law: a = Traction Force/m = Torque x GR/Mcurb

See pg 3 for Section on **Traction Control**.
Then the Torque required to get to g = 1.1, requires that Torque be at least:

Torque_max_g = Weight x km x 1.1 g tire radius/GR = **680 ft lbf**

The present max Torque spec is **713 ft lbf.** This is more than sufficient to give 1.1 g.

$r_{tire} := 14in$

$GR := 9.73$

$M_{curb} := 4936lbm$

$k_m := 1.0447$

$$T_{max\_g} := M_{curb} \cdot k_m \cdot \frac{1.1g \cdot r_{tire}}{GR}$$

$T_{max\_g} = 680.13 \cdot ft \cdot lbf$

$T_{max} := 713ft \cdot lbf$

**What is the Estimated Motor Power needed to meet the 0 - 60 mph in 2.8 s performance, $P_{Spec}$?**

**Ludicrous Mode Specs:** The current has been increased from 1300 to 1500 amps (15 %), the total dual motor power increased from 682 hp to **762 hp** (12%), and the total torque from 707 to 713 ft lbf.
However, as we saw in section I, the System Power to the motor @81% System Efficiency is limited to **649 hp**

$Power_{max} := 762hp$

$$RPM_{motor} := \frac{Power}{Torque \cdot 2 \cdot \pi}$$

There is a basic relationship between Torque, Motor RPM, and Motor Power:
Assume that there is No Traction Control, this is tires can slip.
Thus initially, full torque is applied to the wheels, until max motor power limits the torque.
Refer to the below plot and examine the Power versus time profile.
The Power is given by: Power = Torque x Angular Velocity, until Max Power is reached.
This is shown in the graph below. Tire velocity, $v_{Pmax}$, to get to max motor power and Torque =
The time to get to max motor power is $t_{Pmax}$. The velocity at which this occurs is $v_{Pmax}$.
There are 2 paths to get to the max power: #1 Tires allowed to slip and #2 Tires do not slip (Pg. 3).
Elon Musk said that the peak acceleration is 1.1 g. Designate the time to reach full power, $t_{Pmax}$.

Velocity at Max Power

$$v_{Pmax} := \frac{Power_{max} \cdot r_{tire}}{T_{max} \cdot GR}$$

$v_{Pmax} = 48.05 \cdot mph$

$t_{Pmax} := 1.46s$

$$Pm(\omega) = T(\omega) \cdot k \cdot 2 \cdot \pi \cdot \omega RPM$$

**What Max Power do we need to meet the 0 - 60 mph in 2.8 s with No Traction Control?**
For a vehicle velocity, v, the **Vehicle** Kinetic Energy, KE, required to get to 60 mph, in units of horsepower seconds (hp-s), is given by:
*A hp-s is the amount of energy one hp does in the time of one second. This energy unit more clearly reveals the needed hp over time to to meet the 0-60 mph in 1.8 second spec.*

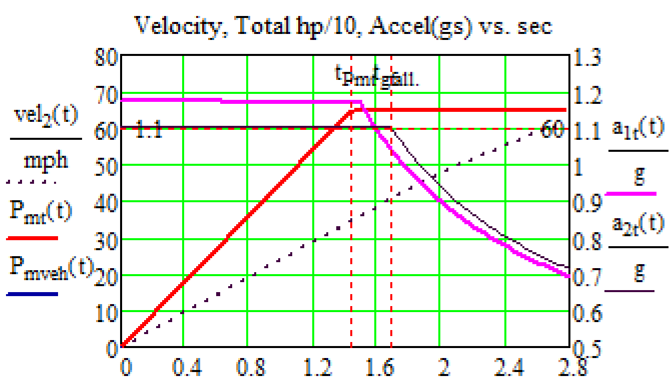
$M_{gross} := M_{curb} + 160lbm$

$$KE(v) := \frac{1}{2} \cdot M_{gross} \cdot k_m \cdot (v)^2$$

$KE(60mph) = 1164.9 \cdot hp \cdot s$

For the graph below, the average **Motor** power from the start at 0 power to the peak of $Power_{max}$ is 1/2 $Power_{max}$. If the time to get motor Power max is $t_{Pmt}$, then the Energy is 1/2 $Power_{max}$ x $t_{Pmax}$. After $t_{Pmax}$ the Motor power is constant. The Energy that goes into the Motor, $E_{motor}$, in *units of horsepower seconds*, is shown below the graph.

**The most critical part** of the model is the 1.11 seconds of Peak Power from the time the acceleration falls from 1.1 g to the 2.8 second spec limit.




The 2 motor power curves belong to two different Traction Models:
#1: No Traction Control, Red Curve
#2: With Traction Control, Blue Curve

$t_{Pmt}$ is the time to Peak Power = 1.46s
$t_{gfall}$ is the time for acceleration(g) to fall below 1.1g = 1.69 s.
These times are in seconds.

$$E_{motor} := \frac{1}{2} \cdot 649hp \cdot 1.46s + 649hp \cdot (2.8 - 1.46)sec$$

$E_{motor} = 1343.43 \cdot hp \cdot s$

**This demonstrates that <u>649 hp is sufficient</u> to meet the 0 - 60 mph in 2.8 sec KE spec. What follows is a more detailed analysis.**

# III. Specifications & Engineering Estimates: Ludicrous Mode Peak Acceleration

System Efficiency: $SysEff := 0.81$ % SOC Voltages: $V_{batt\_100} := 398.4volt$ $V_{batt\_80} := 384volt$ $V_{batt\_50} := 360volt$ $V_{batt} := V_{batt\_100}$
Results are Shown for **100% State of Charge**.
Battery and System Power **@100% SOC** $Power_{Batt} := V_{batt} \cdot 1500A = 801.39 \cdot hp$ $Power_{System} := Power_{Batt} \cdot SysEff = 649.13 \cdot hp$
85 kW-hr Battery
De-acceleration Battery Energy Regeneration Factor: $Regen := 0.64$

**New Algorithm** for Rear to Front Motor Torque & Power Split to maintain **peak acceleration**: $= T_{Split} = 503/259 \sim 1.94$ Gear Ratio:

**Ludicrous Mode Max Power:** $Power_{max} := 649 \cdot hp$ $RPM_{max} := 18000$ $T_{Split} := 1.94$ $GR := 9.73$

**Limit:** $Power_{System} = 649.13 \cdot hp$ $Power_{max} := Power_{System}$ Battery Energy: $Energy_{bat} := 85 \cdot kW \cdot hr$ $R_{phase} := 0.006 ohm$

Max Dual Motor Torque: $Power_{max} = 649.13 \cdot hp$
From TeslaMotors.com/models
$Torque_{maxOld} := 707 \cdot ft \cdot lbf$ $T_{max} := 713 \cdot ft \cdot lbf$

245/35R19 Tire Radius: $r_{tire} := \frac{26.3}{2} in$ $F_{Motor\_Max} := \frac{T_{max} \cdot GR}{r_{tire}}$
Ultra High Performance Tire

Tire Coefficient of Friction, μ: $\mu := 1.1$ $car_{max\_g} := \mu \cdot g$ $k := 1000$ $\tau := 1 \cdot sec$
Max 1.1 g Quoted by Elon Musk

Curb Weight: $M_{curb} := 4936 lbm$ $M_{gross} := M_{curb} + 160 lbm = 5096 \cdot lbm$

Aerodynamic Drag Coeff (TM): $Cd := 0.22$ Average Wind Velocity: $Vw := 0 \cdot mph$ $g_{max} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire} \cdot g}$

Cross Wind Drag Coff: $Cd_{cw} := 0.000014$ Effective Cross Wind V: $V_{cw} := 0 \cdot mph$

Shape Correction Factor: $SCF := 0.85$ Vehicle Frontal Dimensions: $Af := (57 - 7.9) in \cdot 77 \cdot in$ $g_{max} = 1.19$

Air Density, tire resistance: $\rho := 1.293 \cdot \frac{gm}{liter}$ Drag Frontal Area $Ad := Af \cdot SCF$ $Ad = 2.07 \cdot m^2$

Road Rolling Resistance: $RR_{road} := 0.007$ Tire Rolling Resist, Hys: $RR_{tire} := 0.011$ $T_{hys} := 0 \cdot \frac{sec}{m}$

Effective Mass Coefficient: $k_m := 1.0447$ **EPA Range Spec** for P85 is 253 miles See page 6 for $EPA\_RangeSim := 251 mile$

# IV. Tire Traction & Control Models: #1 Perfect Grip, #2 Tires slip, #3 No Slip, #4

**Simple Step Model of Tire Traction (Assume perfect weight distribution per motor, i.e. same acceleration at each motor)**
Depending on road conditions, Tires do not have perfect grip, they may slip. Vehicle acceleration, $a_{veh}$ is limited to the maximum tire traction ($tire_{max\_g}$) = 1.1g. The tire rpm x GR = motor rpm, but because of slip, tire velocity can be greater than vehicle velocity. Therefore, vehicle acceleration and velocity are not directly proportional to rpm, that is, tires may slip:

**Case #1** $v_{tire} = v_{vehicle}$
**No Traction Control, but no tire slip.**
Max motor power and torque are applied to tires. Perfect Tires that do not slip. Acceleration can **exceed 1.1 g**.

**Case #2 - Ludicrous Mode** $v_{tire} = v_{vehicle}$
**Perfect Traction System and High Performance g = 1.1 tires.**
Because of Traction Control and tire slip, effective motor rpm can be greater than vehicle speed during tire slip or Traction Control. Vehicle speed depends on **tire** coefficient of resistance, μ, which is equal to 1.1 for Ludicrous. This allows a **max of 1.1 g**. For Case #2, we assume Traction Control limits g to 1.1.

# Macro Model of Motor Dynamics: Velocity of Tire is v

Angular Velocity Symbol, Ω (units of radians/second) $\Omega(\omega) := 2\pi 1000 \cdot \omega \cdot min^{-1}$ RPM/1000 Symbol, $\omega_k$ $RPM := min^{-1}$

Angular Vel Ω @Max Power: $\Omega_{Pmax} := Power_{max} \cdot T_{max}^{-1}$ $RPM_{Pmax} := \frac{\Omega_{Pmax}}{2 \cdot \pi}$ $RPM_{Pmax} = 4781.63 \cdot RPM$

Convert velocity to RPM $VtoRPM(v_v) := v_v \cdot (1000 \cdot 2 \cdot \pi \cdot r_{tire} \cdot RPM)^{-1}$ $\omega_{Pfall} := RPM_{Pmax} \cdot k^{-1} = 4.78 \cdot RPM$

Tire Velocity at Torque Fall: $v_{Tfall} := RPM_{Pmax} \cdot 2 \cdot \pi \cdot r_{tire} \cdot GR^{-1}$ $v_{Tfall} = 38.45 \cdot mph$

Tire Velocity to kRPM: $VtokR(v_t) := v_t \cdot (k \cdot 2 \cdot \pi \cdot r_{tire} \cdot RPM)^{-1}$ $VtokR(60 \cdot mph) \cdot GR = 7.46$

Road Resistance, Ft: $Ft(v_v) := M_{gross} \cdot g \cdot \left[T_{hys} \cdot v_v \cdot \sin(\theta) + (RR_{tire} + RR_{road}) \cdot \cos(\theta) + \sin(\theta)\right]$ $RPM_{pmax}$ for Max Power:

Air Drag Force, Fa: $Fa(v_v) := 0.5 \cdot \rho \cdot Ad \cdot \left[(v_v + Vw)^2 \cdot Cd + Cd_{cw} \cdot (V_{cw})^2\right]$

**Note:** For Drag and Road Resistance, approximate vehicle with $v_{tire}$. At <60mph Compared to Ftire, Fo is small.

Total Opposing Force, Fo: $Fo(v_v) := Fa(v_v) + Ft(v_v)$ $Fo(60 \cdot mph) = 139.42 \cdot lbf$

Torque/Force Falloff Curve: $\omega_{kmax} := 15.8 \cdot RPM$ $T_{PLt}(\omega_k) := Power_{max} \cdot \Omega(\omega_k)^{-1}$ $T_{PLt}(55) = 61.99 \cdot ft \cdot lbf$

Tm is Torque of motor $T_m(\omega_k) := if(\omega_k \cdot RPM \ge \omega_{Pfall}, T_{PLt}(\omega_k), T_{max})$ $P_m(\omega_k) := T_m(\omega_k) \cdot k \cdot 2 \cdot \pi \cdot \omega_k \cdot RPM$
Fmot, Tractive Force from motor, not from slipping tires: $T_{mv}(v_t) := T_m(VtokR(v_t) \cdot GR)$ $F_{mot}(v_t) := \frac{GR}{r_{tire}} \cdot T_{mv}(v_t)$ $F_{PL}(v_t) := Power_{max} \cdot (v_t \cdot mph)^{-1}$

# Solve for Velocity, Acceleration, and Distance versus Time

**We are using Mathcad 14, a Computer Math Program, to do the Calculations: http://www.ptc.com/product/mathcad/free-trial**

## Case 1: Perfect Grip Tires at Maximum Motor Power, Coefficient of Tire Friction > 1.1 g

**Newton's Third Law of Motion:**

$$a_1(v) := \frac{F_{mot}(v) - Fo(v)}{k_m \cdot M_{gross}} \qquad a_{1Tmax} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire}} = 1.19 \cdot g$$

$$V := 0 \cdot mph \qquad vel_1(t) := root\left( t \cdot sec - \int_0^V \frac{mph}{a_1(V \cdot mph)} dV, V \right) \cdot mph \qquad time_{a1}(v) := \int_0^v \frac{1}{a_1(v)} dv$$

$time_{a1}(60mph) = 2.58\ s$

$vel_1(2.64) = 60.91 \cdot mph$

$$v_{gfall} := root\left(a_1(V \cdot mph) - car_{max\_g}, V\right) = 40.77 \qquad time_{a1}(60mph) = 2.58\ s \qquad a_{1t}(t) := a_1\left(vel_1(t)\right)$$

Velocity g fall, $a \le 1.1g$ $\qquad a_1\left(v_{gfall}\, mph\right) = 1.1 \cdot g \qquad a_{1t}(0) = 1.17 \cdot g$

## Case 2-LudicrousMode: High Performance 1.1 g Tires & Motor Drive Limited Accel < 1.1 g/ No Spin, but Max Power

$a_2$ acceleration is allowed by high performance tires on dry road.

$$a_2(v) := if\left(a_1(v) \ge car_{max\_g}, car_{max\_g}, a_1(v)\right) \qquad car_{max\_g} = 1.1 \cdot g$$

$$vel_2(t) := root\left( t \cdot sec - \int_0^V \frac{mph}{a_2(V \cdot mph)} dV, V \right) \cdot mph \qquad time_{a2}(v) := \int_0^v \frac{1}{a_2(v)} dv$$

$vel_2(2.8) = 61.87 \cdot mph$

$a_2\left(v_{gfall}\, mph\right) = 1.1 \cdot g$

$$distance_2(t) := \int_0^t vel_2(t)\, \tau\, dt \qquad a_{2t}(t) := a_2\left(vel_2(t)\right) \qquad a_{2t}(0mph) = 1.1 \cdot g$$

$time_{a2}(60mph) = 2.68\ s$

$distance_2(10.5) = 0.25 \cdot mile \qquad t_{gfall} := time_{a2}\left(v_{gfall} \cdot mph\right) = 1.69\ s$ $\qquad$ RPM at g fall: $\quad R_{gfall} := VtokR\left(v_{gfall} \cdot mph\right) \cdot GR$

## Case 3: Traction Control - Tire Force is Power Limited - No Tire Spin (This model not yet perfected)

$$F_{1\_1g} := k_m \cdot M_{gross} \cdot car_{max\_g} = 5856.17 \cdot lbf \qquad T_{1\_1g} := \frac{F_{1\_1g} \cdot r_{tire}}{GR} \qquad T_3(\omega_k) := if\left(\omega_k \le R_{gfall}, T_{1\_1g}, Power_{max} \cdot \Omega(\omega_k)^{-1}\right)$$

$$P_{1\_1g}(\omega_k) := T_{1\_1g} \cdot \omega_k \cdot k \cdot 2 \cdot \pi \cdot RPM \qquad P_{1\_1g}(5.252) = 659.53 \cdot hp \qquad \omega_{3Pmax} := 5252$$

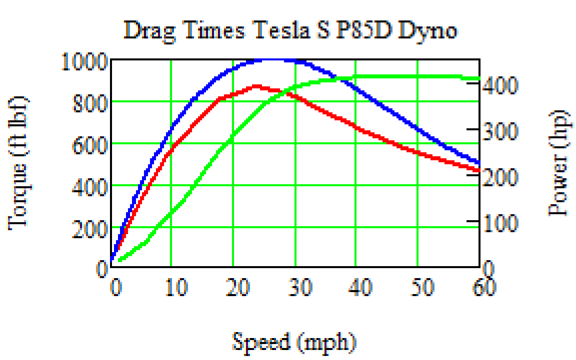
$$P_3(\omega_k) := T_3(\omega_k) \cdot \left(\omega_k \cdot k \cdot 2 \cdot \pi \cdot RPM\right) \qquad F_3(v) := \frac{GR}{r_{tire}} \cdot T_3(VtokR(v \cdot mph) \cdot GR)$$

**Case 3:** We end up getting the same effective peak torque, we just don't waste the power put into spinning wheels. Tesla has a patent on how to split the power

## Case 4: Fit Torque to Drag-Times Dyno Torque Curve Shape -->Even @100% Efficiency, Does Not Meet Specs

**$a_4(v)$ = P85D Acceleration/Torque, $a_1(v)$ x $T_{Shape}(v)$**
**$T_{Shape}(v)$ was fitted to have the same shape,** as the **Drag Times Torque/speed Curve - See Graph Below**

$$a_4(v) := if\left(a_1(v) \cdot T_{Shape}\left(\frac{v}{mph}\right) \ge car_{max\_g}, car_{max\_g}, a_1(v) \cdot T_{Shape}\left(\frac{v}{mph}\right)\right)$$




**Dyno Data by Drag Times for 2015 Tesla S P85D**
Dyno Torque (Red) and Dyno Power (Green)
DynoTorque Shape **x 1000** (Blue), Extracted from Dyno Torque
**http://www.dragtimes.com/2015-Tesla-Model-S-Videos-27143.html**

$$vel_4(t) := root\left( t \cdot sec - \int_0^V \frac{mph}{a_4(V \cdot mph)} dV, V \right) \cdot mph$$

$$vel_{4.}(t) := vel_4(t) \cdot mph^{-1}$$

$$a_{4t}(t) := a_4\left(vel_2(t)\right) \qquad time_{a4}(v) := \int_0^v \frac{1}{a_4(v)} dv$$

**Does not meet Specs**

$time_{a4}(50mph) = 4.24\ s$

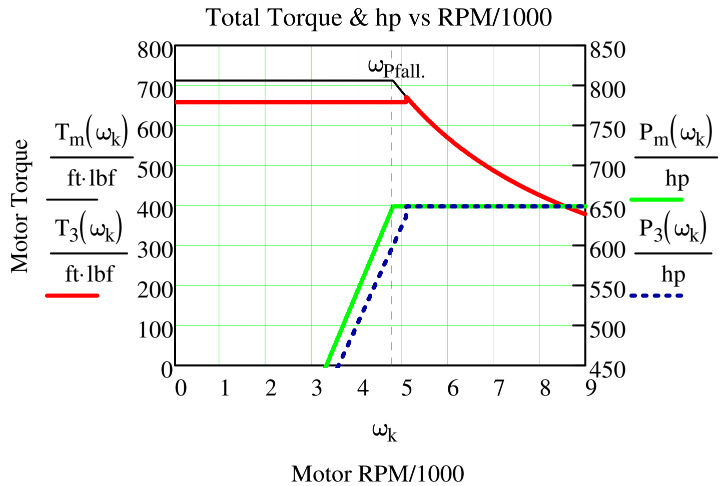
# V. Validation: The Model and the Estimate of System Parameters Meet the Ludicrous Specs

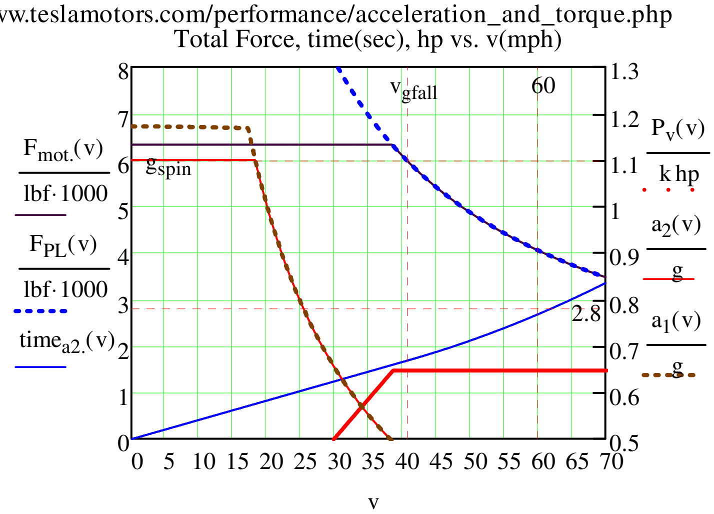
$time_{a2}(60 \cdot mph) = 2.68\ s \qquad distance_2(10.46) = 0.24 \cdot mile$

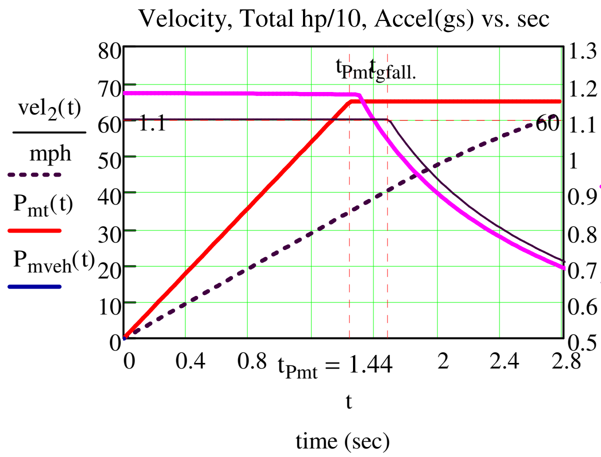
Calculated P90 EPA Range: 230 Miles

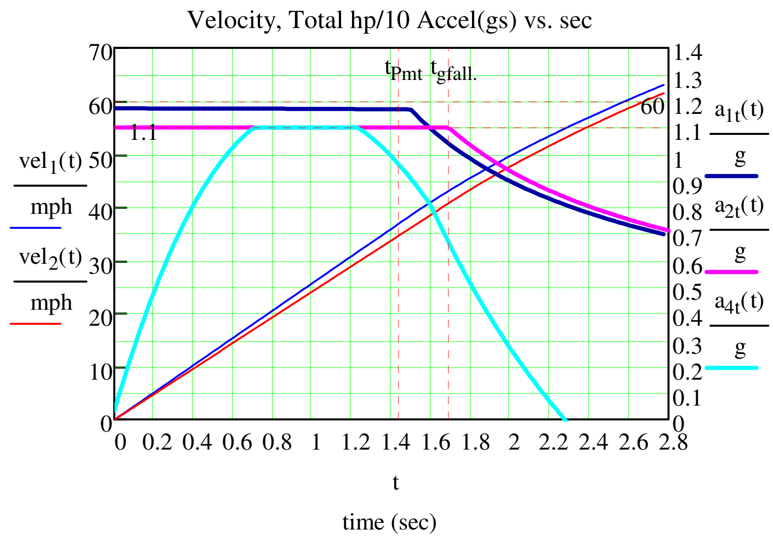
## VI. Graphs

Compare with Torque & Power Curves at: http://www.teslamotors.com/performance/acceleration_and_torque.php



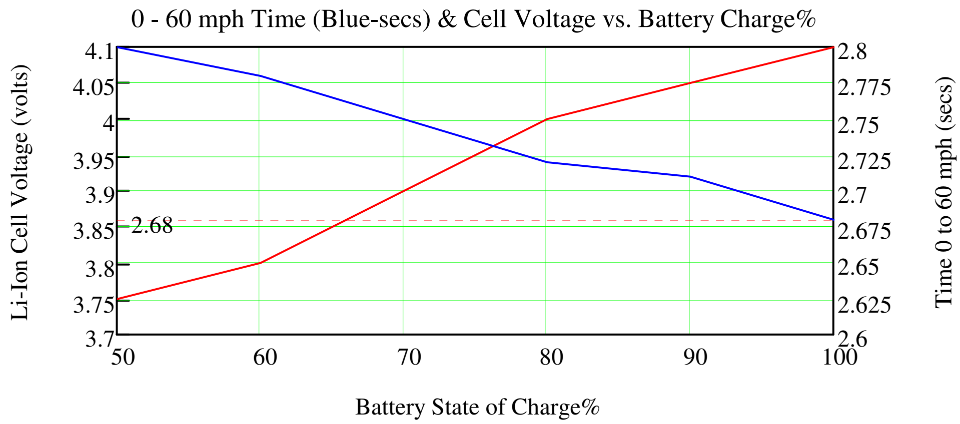




## Time 0-60 mph (Blue-secs) versus Battery State of Charge, SOC%

Time Meets Specs from 100 to 50% SOC




**Panasonic Li-Ion P18650**
**Approximate Cell Voltage Data:**
http://www.teslamotorsclub.com/showthread.php/44691-P85D-691HP-should-have-an-asterisk-*-next-to-it-Up-to-691HP/page5?p=948071#post948071

## VII. Find the Single Charge. Highway Cruise Range for a Given Velocity and Final SOC

**Driving Pattern/Profile:** Assume we cruise at constant speed, but start, stop, and regen break four times per hour

**Drive Train Power Efficiency - Battery Loss for Commanded Vehicle Velocity and Final State of Charge, $SOC_f$:**

**$SOC_f$ is 10% at recharge.** 400V HV battery idle power is Po. 12V battery gives Accessory Power. The Traction Inverter Efficiency - TInvE, HV Power Electronics at Idle Efficiency - IPEE, and Gear Power Efficiency - GPE are 92.5%, 95%, and 90%, respectively. Brake Regen efficiency of kinetic energy is 64%. Then the number of starts per hour as a function of velocity, NS, NumStarts(v, Po), is

Change in State of Charge = 1 - $SOC_f$

$TInvE := 0.925 \qquad IPEE := 0.95 \qquad GPE := 0.9 \qquad Regen := 0.64$

$$Power_{dissLoss}(v, P_o) := \frac{Fo(v)\cdot v}{TInvE\cdot GPE} + \frac{P_o\cdot watt}{IPEE} \qquad Energy_{accel}(v) := Power_{max}\cdot time(v\cdot mph)\cdot hr$$

NSo, NS are iterative converging estimates of total NumStarts per charge

$$NS_o(v) := 2\cdot\left[\frac{65mph}{(v + 0.1\cdot mph)}\right]^2 \qquad NS\left(v, P_o, SOC_f\right) := \frac{Energy_{bat}\cdot\left(1 - SOC_f\right) - NS_o(v)\cdot\left[\left[\frac{M_{gross}\cdot(v)^2}{2}\cdot(1 - Regen)\right]\right]}{Power_{dissLoss}\left(v, P_o\right)\cdot 15\cdot min}$$

$$NumStarts\left(v, P_o, SOC_f\right) := floor\left[\frac{Energy_{bat}\cdot\left(1 - SOC_f\right) - NS\left(v, P_o, SOC_f\right)\cdot\left[\frac{M_{gross}\cdot(v)^2}{2}\cdot(1 - Regen)\right]}{Power_{dissLoss}\left(v, P_o\right)\cdot 15\cdot min}\right]$$

$$Cruise\_Range\left(v, P_o, SOC_f\right) := \frac{\left[Energy_{bat}\cdot\left(1 - SOC_f\right) - NumStarts\left(v, P_o, SOC_f\right)\cdot\left[\frac{Regen\cdot M_{gross}\cdot(v)^2}{2}\cdot(1 - Regen)\right]\right]\cdot v}{Power_{dissLoss}\left(v, P_o\right)}$$

## Highway Cruise Range with Four Stops per Hour Estimate

$Cruise\_Range(30\cdot mph, 100, 0.1) = 302.55\cdot mi$

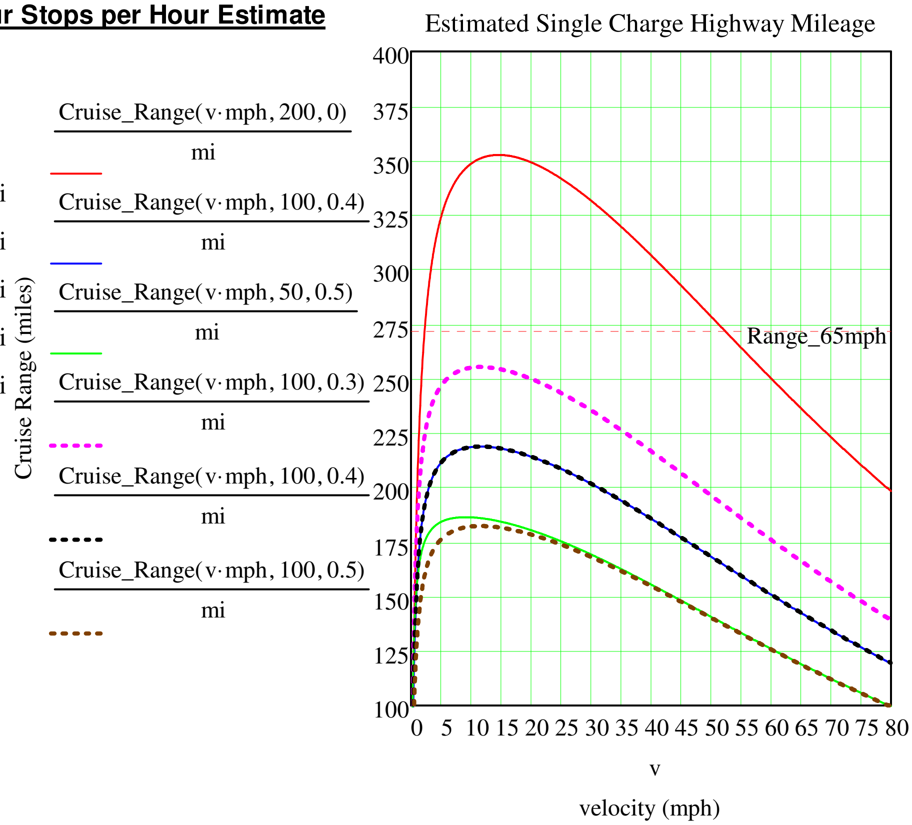
$Cruise\_Range(40\cdot mph, 100, 0.1) = 278.53\cdot mi$

$Cruise\_Range(50\cdot mph, 100, 0.1) = 252.27\cdot mi$

$Cruise\_Range(60\cdot mph, 100, 0.1) = 226.28\cdot mi$

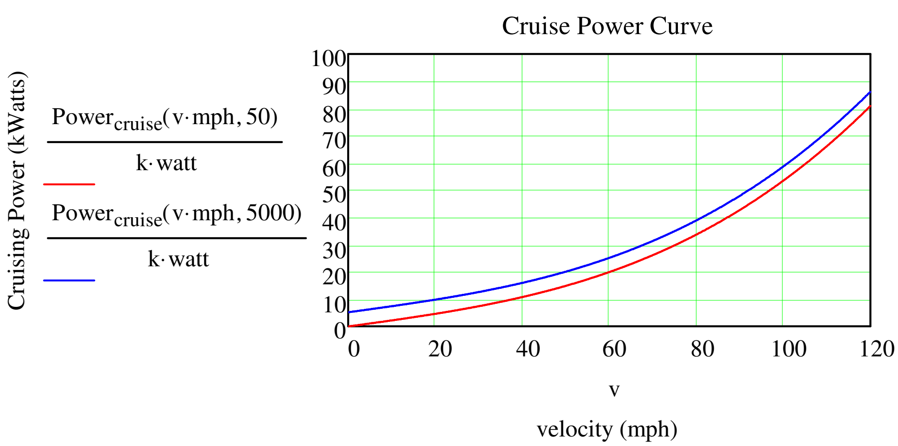
$Cruise\_Range(70\cdot mph, 100, 0.1) = 201.52\cdot mi$

$Cruise\_Range(60\cdot mph, 200, 0) = 250.04\cdot mi$




## Opposing Force (Air Resistance, Tire, Road Resistance) Power Loss

$Power_{cruise}\left(v, P_o\right) := Power_{dissLoss}\left(v, P_o\right) \qquad Power_{cruise}(0\cdot mph, 500) = 0.71\cdot hp$



# VIII. Find Mileage Range: Use 3 Different EPA Driving Schedules

**Algorithm to Calculate Range, Range(P,fHz), 100% Battery Discharge, Driving Profile Velocity/Time File, P and Sampling Rate, fHz**

$Energy_{bat} = 85 \cdot kW \cdot hr$

$$
Range(P, f_{Hz}) := \left|
\begin{array}{l}
Ebat \leftarrow E_{diss} \leftarrow v_{old} \leftarrow 0 \\
n \leftarrow -1 \\
N \leftarrow rows(P) - 1 \\
\text{while } \left( E_{diss} < \dfrac{Energy_{bat}}{kW \cdot hr} \right) \\
\quad \left|
\begin{array}{l}
n \leftarrow n + 1 \\
t \leftarrow mod(n, N) \\
v \leftarrow P_t \\
P_{accel} \leftarrow \dfrac{k_m \cdot M_{gross} \cdot \left(v^2 - v_{old}^2\right) \cdot \dfrac{mph \cdot f_{Hz}}{sec} mph}{TInvE \cdot GPE \cdot 2} \quad \text{if } v > v_{old} \\
P_{accel} \leftarrow k_m \cdot M_{gross} \cdot \left(v^2 - v_{old}^2\right) \cdot \dfrac{mph \cdot f_{Hz}}{2 sec} mph \cdot Regen \quad \text{otherwise} \\
E_{diss} \leftarrow E_{diss} + \dfrac{\left(Power_{dissLoss}(v \cdot mph, 100) + P_{accel}\right) \cdot sec}{kW \cdot hr \cdot f_{Hz}} \\
v_{old} \leftarrow v \\
Ebat_n \leftarrow E_{diss}
\end{array}
\right. \\
Range \leftarrow \displaystyle\sum_{m=0}^{n} \dfrac{\left(P_{mod(m,N)} + P_{mod(m+1,N)}\right) \cdot mph \cdot sec}{2 \cdot mi \cdot f_{Hz}}
\end{array}
\right.
$$

If decelerating, charge battery with Regen fraction of energy.

**Read US06 and FTP Dynamometer Drive Profile Files**

**Refer to**: http://www.epa.gov/nvfel/testing/dynamometer.htm

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds. Sampling can be either 1 Hz or 10Hz

The **Federal Test Procedure (FTP)** is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. 10 Hz Sampling data is named FP10 and HY10 for the Highway schedule.

FTPF := READPRN("FedTestProc.txt") $\quad t := FTPF^{\langle 0 \rangle} \quad FTP := FTPF^{\langle 1 \rangle} \quad rows(FTP) = 1875$

UDDSF := READPRN("uddscol.txt") $\quad UDDS := UDDSF^{\langle 1 \rangle} \quad rows(UDDS) = 1370$

HWYF := READPRN("hwycol.txt") $\quad HWY := HWYF^{\langle 1 \rangle} \quad R_{hwy} := rows(HWY)$

FP10 := READPRN("FTP10Hz.TXT") $\quad FTP10V := submatrix(FP10, 0, rows(FP10) - 1, 1, cols(FP10) - 1)$

HY10 := READPRN("HWY10Hz.TXT") $\quad HWY10V := submatrix(HY10, 0, rows(HY10) - 1, 1, cols(HY10) - 1)$

US06F := READPRN("US06PROFILE.TXT") $\quad Time := US06F^{\langle 0 \rangle} \quad US06 := US06F^{\langle 1 \rangle} \quad n_6 := 0 .. 598$

$r1 := 0 .. rows(HY10) \cdot 10 - 1 \quad HWY10_{r1} := HWY10V_{ceil\left(\frac{r1+1}{10}\right)-1,\ mod(r1, 10)}$

## Using EPA Profiles and above Range Program, Calculate Tesla EV Range for EPA Profiles

$Range_{US06} := Range(US06, 1) \quad Range_{FTP} := Range(FTP, 1) \quad Range_{HWY} := Range(HWY, 1)$

**EPA 2008 Cycle MPG Fuel Economy Least Squares Fit Regression for Range**

$$MPG_{city} := \frac{1}{\left(0.003259 + \frac{1.18053}{Range_{FTP}}\right)} \qquad MPG_{hwy} := \frac{1}{0.001376 + \frac{1.3466}{Range_{HWY}}}$$

$$MPG_{epa} := 0.55 \cdot MPG_{city} + 0.45 \cdot MPG_{hwy}$$

## Single Charge EPA Range Calculations: Federal Test Procedure (FTP), Highway, and US06

**Model Validation:** **Published EPA Range is 260 miles**

$Range_{FTP} = 229.44 \qquad Range_{HWY} = 245.36 \qquad Range_{US06} = 180.11$

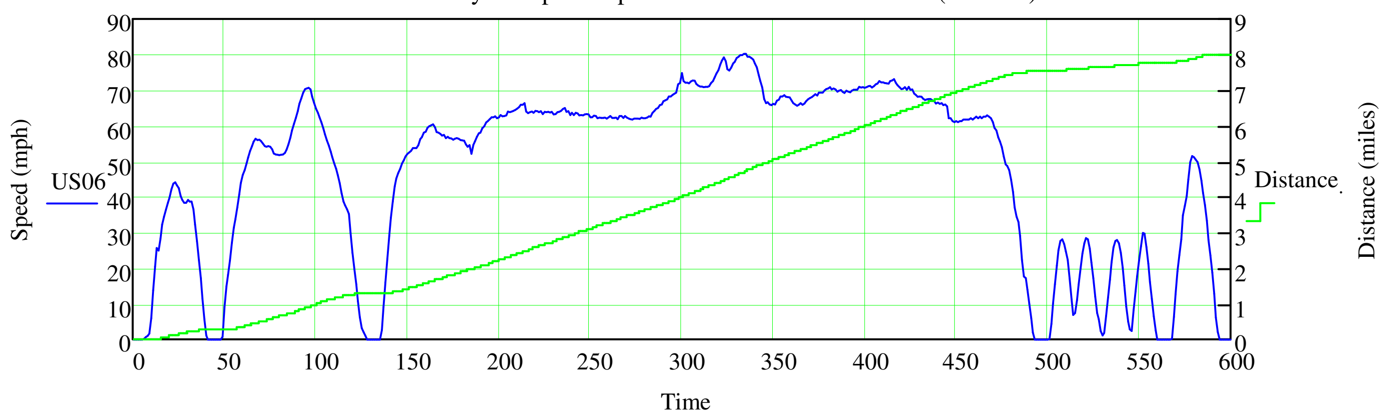
$MPG_{city} = 118.99 \qquad MPG_{hwy} = 145.68 \qquad MPG_{epa} = 131$

$r := 0 .. rows(FTP) - 1 \qquad Distance_r := \sum_{r=0}^{r} FTP_r \cdot \frac{1}{60 \cdot 60} \qquad rr := 0 .. rows(US06) - 1 \qquad Distance._{rr} := \sum_{rr=0}^{rr} US06_{rr} \cdot \frac{1}{60 \cdot 60}$

$max(Distance) = 11.04 \qquad max(Distance) = 11.04 \qquad max(Distance.) = 8.01$

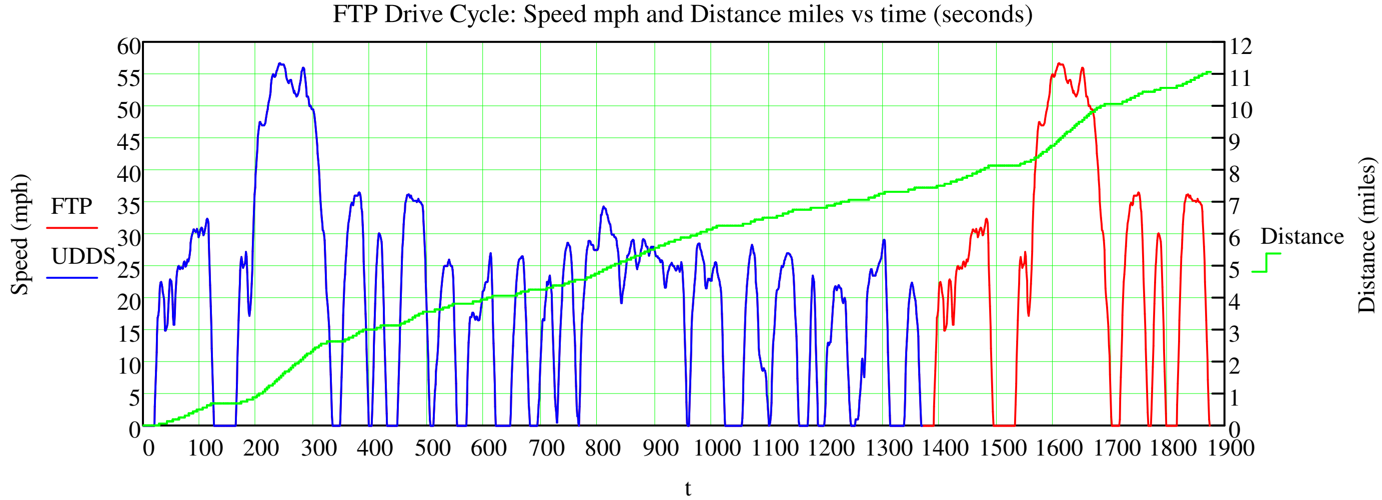
**Plots of EPA Dynamometer Vehicle Testing Profiles**

US06 Drive Cycle: Speed mph and Distance miles vs time (seconds)




Time (seconds)

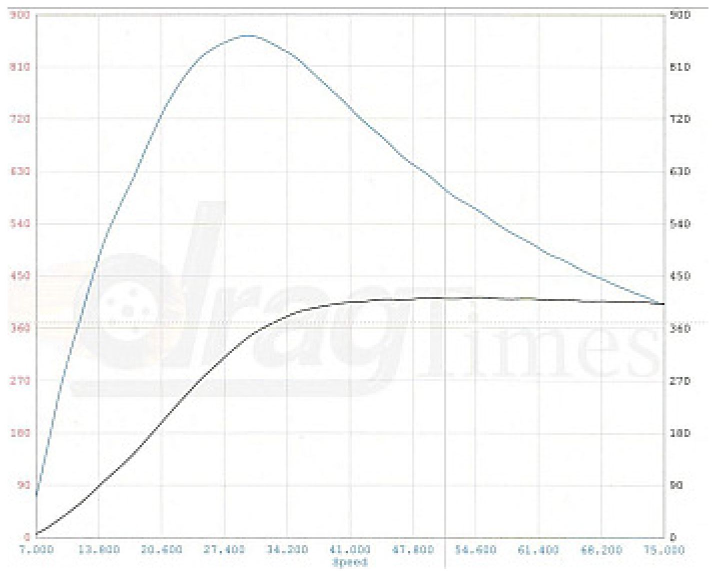
FTP Drive Cycle: Speed mph and Distance miles vs time (seconds)




Time (seconds)

# X. Drag Times Tesla Torque and Power Dynamometer

**http://www.dragtimes.com/2015-Tesla-Model-S-Videos-27143.html**




**Extract Points from Curves to csv data files**

**Read Dyno Data csv data files**

$T_{Dyn} :=$ READPRN("Tesla P85D DynoTorq Insane.csv") $\quad T_{Dyn}^{\langle 0\rangle} := T_{Dyn}^{\langle 0\rangle} - 5.916 \quad \mathrm{rows}(T_{Dyn}) = 27$

$P_{Dyn} :=$ READPRN("Tesla P85D DynPowerR.csv") $\quad P_{Dyn}^{\langle 0\rangle} := P_{Dyn}^{\langle 0\rangle} - 5.916 \quad \mathrm{rows}(P_{Dyn}) = 24$

**Power Fit Torque to get Shape**

Guess: $a := 1 \quad b := 1 \quad c := 1 \quad d := 27 \quad e := 1 \quad f := 30 \quad gx := 0$

Given $\quad Torq := T_{Dyn}^{\langle 1\rangle} \quad spd := T_{Dyn}^{\langle 0\rangle}$

$Torq - \left[a\cdot(spd)^3 - c\cdot(spd - d)^2 - e\cdot(spd - f) + gx\right] = 0 \qquad At := \mathrm{Minerr}(a, c, d, e, f, gx)$

$\mathrm{Torque}(s) := At_0\cdot(s)^3 - At_1\cdot\left(s - At_2\right)^2 - At_3\cdot\left(s - At_4\right) + At_5 \qquad \mathrm{Torque}(0) = 23.24$

Guess $\quad a := 1 \quad b := 1 \quad c := 1 \quad d := 1 \quad e := 1 \quad f := 30 \quad gx := 0 \quad h := 0 \quad i := 0$

Given $\quad P_{wheel} := P_{Dyn}^{\langle 1\rangle} \quad spd := P_{Dyn}^{\langle 0\rangle} \quad n := 0..159 \quad S_n := n\cdot 0.5 \quad Tx_n := \mathrm{Torque}(S_n)$

$P_{wheel} - \left[a\cdot(spd - b)^3 - c\cdot(spd - d)^2 - e\cdot(spd - f) + gx\cdot(spd - h) + i\right] = 0 \qquad A1 := \mathrm{Minerr}(a, b, c, d, e, f, gx, h, i)$

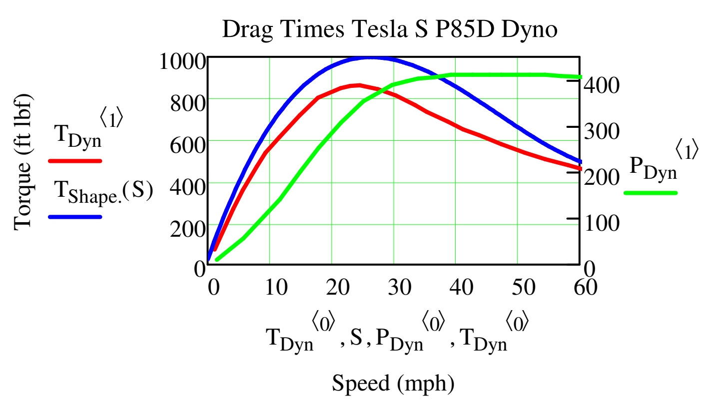
$\mathrm{Pwr}(s) := A1_0\cdot\left(s - A1_1\right)^0 - A1_2\cdot\left(s - A1_3\right)^2 - A1_4\cdot\left(s - A1_5\right) + A1_6\cdot\left(s - A1_7\right) + A1_8 \qquad \mathrm{Pwr}(30) = 344.2$

$P_{Dyn\_Fv} := \overrightarrow{\left(T_{Dyn}^{\langle 1\rangle}\frac{ft\cdot lbf}{r_{tirex}\cdot in}\cdot T_{Dyn}^{\langle 0\rangle}\cdot\frac{mph}{14\cdot hp}\right)} \qquad {P_{Dyn\_Fv}}_{20} = 177.41\ m^{-1}$

**Normalize Torque Curve to Max = 1 to extract shape only, TShape**

Torque Shape: $\quad NormT := \frac{1}{\max(Tx)} \qquad T_{Shape}(s) := \mathrm{Torque}(s)\cdot NormT \qquad T_{Shape}(s) := T_{Shape}(s)\cdot 1000$

Multiple by 1000 to plot on same axis with Original Torque Curve




$$At = \begin{pmatrix} 0.01 \\ 1.88 \\ 7.86 \\ -41.57 \\ -154.22 \\ -6270.96 \end{pmatrix} \qquad A1 = \begin{pmatrix} 0 \\ 6.65 \\ 0.36 \\ -21.66 \\ 9.2 \\ 278.18 \\ 49.07 \\ 18.22 \\ -1557.66 \end{pmatrix}$$

# IX. Tire Friction (Composition and Width)

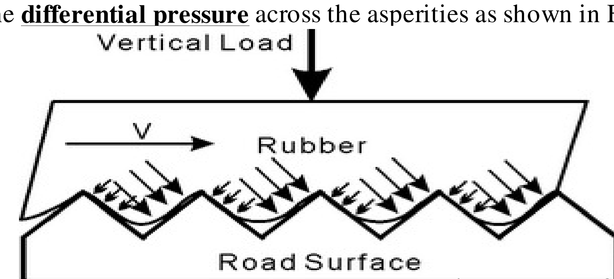
**Coefficient of Static Friction (μ) is the ratio of Tire Road Force to Vehicle Weight.** Values of μ for Conventional Car tire On: Asphalt 0.72, Car tire Grass 0.35.

**Top Fuel drag car tires** are getting a coefficient of friction well **over 4.5. How is this possible?**

**This material came from: http://insideracingtechnology.com/tirebkexerpt1.htm** **See Mathcad/EVs/Tire Friction.doc**

**Rubber generates friction** in three major ways: **adhesion, deformation, and wear.**

Rubber in contact with a **smooth surface** (glass is often used in testing) generates friction forces mainly by **adhesion**. **When rubber is in contact with a rough surface, another mechanism, deformation, comes into play. Movement** of a rubber slider on a r**ough surface** results in the **deformation of the rubber by high points o**n the surface called irregularities or **asperities**. A load on the rubbe slider causes the asperities to **penetrate the rubber** and the **rubber drapes over** the asperities. The **energy needed** to move the asperities in the rubber comes from the **differential pressure** across the asperities as shown in Fig. 3.4, where a rubber slider moves on an irregular surface at speed V.




**Tearing and Wear**

As deformation forces and sliding speeds go up, **local stress** can e**xceed the tensile strength of the rubber**, especially at an increase in local stress near the point of a **sharp irregularity**. High local stress can **deform the internal structure of the rubber past the point of elastic recovery.** When polymer bonds and crosslinks are **stressed to failure the material can't recover completely**, and this can cause **tearing. Tearing absorbs energy**, resulting in **additional friction forces** in the contact surface.

Wear is the ultimate result of tearing.

Ftotal = Fadhesive + Fdefformation + Fwear

**Deformation Friction and Viscoelasticity**

**Rubber is elastic** and conforms to surface irregularities. But rubber is **also viscoelastic**; it **doesn't rebound fully** after deformation.

**Hysteresis**

Hysteresis, or energy loss, in rubber.
where there is **some sliding** between the rubber and an irregular surface. If the **rubber recovers slowly** from the passing irregularity as in the high-hysteresis rubber, it **can't push o**n the downstream surfaces of the irregularities **as hard** as it pushes on the upstream surfaces. This **pressure differenc**e between the **upstream and downstream faces of the irregularity** results in **friction forces** even when the surfaces are lubricated.

**Wide Tires:** It is true that wider tires commonly have better traction. The main reason why this is so does not relate to contact patch, however, but to **composition. Soft compound tires** are required to be **wider in order for the side-wall to support the weight** of the car softer tires have a larger coefficient of friction, therefore better traction. A narrow, soft tire would not be strong enough, nor would it last very long. **Wear in a tire is related to contact patch. Harder compound tires wear much longer**, and can be narrower. They do, however have a lower coefficient of friction, therefore less traction. Among tires of the same type and composition, here is no appreciable difference in 'traction' with different widths. **Wider tires**, assuming all other factors are equal, commonly have **stiffer side-walls and experience less roll. This gives better cornering performance.**

Friction is proportional to the normal force of the asphalt acting upon the car tires. This force is simply equal to the weight which is distributed to each tire when the car is on level ground. Force can be stated as Pressure X Area. **For a wide tire, the area is large but the force per unit area is small** and vice versa. The force of friction is therefore the same whether the tire is wide or not. However, **asphalt is not a uniform surface**. Even with steamrollers to flatten the asphalt, the surface is still somewhat irregular, especially over the with of a tire. Drag racers can therefore **increase the probability or likelihood of making contact** with the road by using a wider tir In addition a secondary benefit is that the wider tire increased the support base a

F**riction force is independent of the apparent area of contact. For hard materials**, this is nearly correct. The true area of contact varies with the applied load. The apparent area does not. If you can imagine the contact zone from a **microscopic viewpoint, only a tiny portion of the apparent area actually touches.** This tiny area is the true area of contact. But this applies to hard materials. It does **not apply to elastomers, such as rubber.** Tire tread rubber compounds vary greatly from one application to another. **Race car tire** tread compounds can be **very soft, viscoelastic materials**, while heavy truck tread rubber can be quite hard. In general, **soft rubber materials have greater friction**. With drag racing 'slicks,' the tire tread material **literally sticks** to the pavement--and the **rubber is sheared from the tire.** Clearly, the greater the apparent contact area, the greater this shear force. **Cleanliness is important** to getting the surfaces to '**stick.**' This is one reason why drag racers have a '**burn-out'** before each race (another is to raise the tire tread surface temperature). However, there is another reason for wide tire treads on some road and track racing cars. They need **tread volume to provide enough wear life**. Tires wear rapidly under racing conditions. **Some long races wear out several sets of tires.** There are **trade-offs with traction and tread life.** That is why heavy truck tire tread compounds do not have as much friction as those used on passenger cars. However, truck tire tread compounds provide longer wear life and less heat build-up. Like many things in this world, **tire tread choices involve compromises.**