

PRIUS GEN III 1.5L PERFORMANCE SIMULATION

<http://www.leapcad.com/Transportation/Prius Gen III 1.5L Simulation.mcd>

Goal: Macro Model the Prius Gen III 1.5L ICE Vehicle Performance

Procedure:

First model the Prius Internal Combustion Engine's (ICE) and motor's, torque and power. Define vehicle and road parameters.

Acceleration Protocol:

Use motor/generator speed, MG2 rpm, ω_{m2} , (axle of planetary gear) as a control variable. Initially, use the high starting torque from motor MG2 to drive the axle. Supply peak electrical power to MG2 from both the battery (peak battery power for 10 seconds) and from MG1 (run as a generator at it's max speed, mechanically driven by the the ICE). This condition simulates a low gear mode for the ICE.

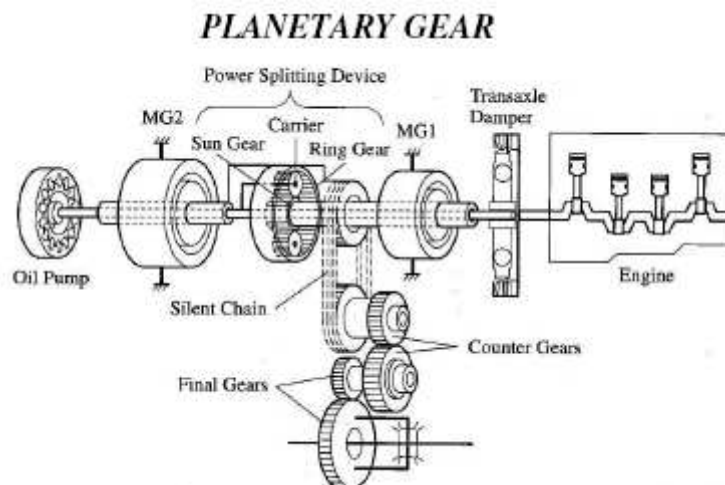
As MG2 speeds up and demands increasing power, throttle back it's drive/torque by limiting its electrical power input to the max Inverter Output, i.e. sum of battery and MG1 generator power. Refer to the curve of M2 Torque vs. ω_{m2} on page 3.

Use equations for vehicle dynamics to calculate the time to 60 mph. Plot performance simulation curves. Calculate All Electric Range, AER, miles from a single charge of the battery. Get AER results using various EPA driving modes.

Basic Description of Prius

Modeling and Simulation of Serial Parallel Hybrid EV

Description of Synergy (Planetary Gear) Drive



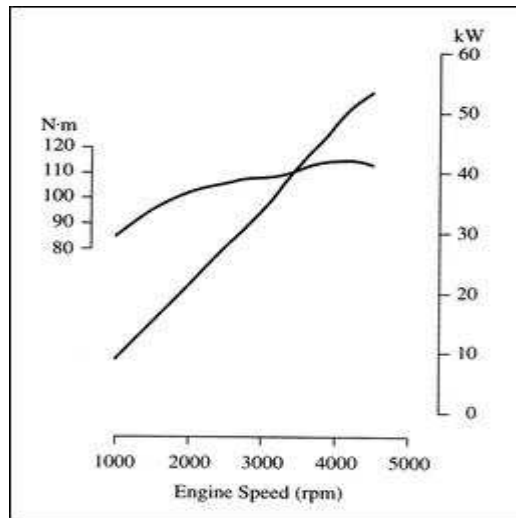
Rotation Speed Variables:

MG1, MG2, and Engine: ω_{m1} , ω_{m2} , and ω_{ice} .

Power Efficiencies:

Inverter, axle to tire, engine to MG1, and engine to axle efficiencies η_{inv} , η_{axle} , η_{eng_m1} , and η_{aeng_axle}

Prius Gen III ICE Performance Curves

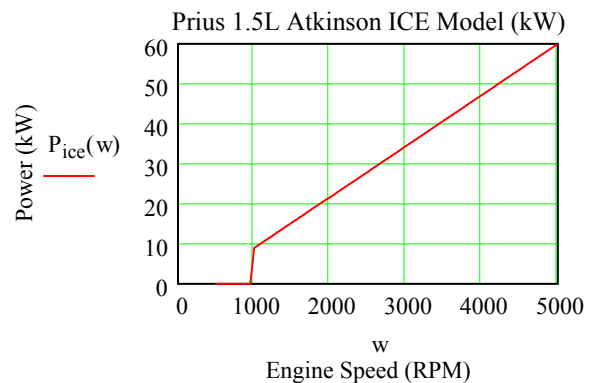
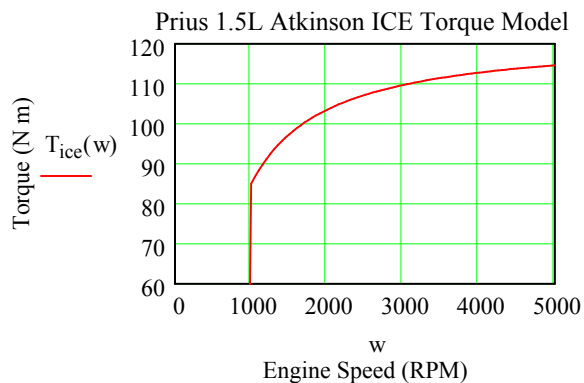


Power and Torque vs. RPM Models of Toyota Atkinson Gen III 1.5L ICE

Match Model Parameters to: [Specifications for Different Prius Models](#)

$$k := 1000 \quad w := 500, 550.. 5000 \quad T_{idle} := 85 \quad \Omega_{idle} := 1000 \quad P_{idle} := T_{idle} \cdot \left(\frac{2 \cdot \pi \cdot \Omega_{idle}}{60 \cdot k} \right) \quad P_{slope} := \frac{57 - 9}{4000}$$

$$P_{ice}(w) := \text{if} \left[w < \Omega_{idle}, 0, P_{idle} + P_{slope} \cdot (w - \Omega_{idle}) \cdot \frac{115}{108} \right] \quad T_{ice}(w) := \left(P_{ice}(w) \right) \cdot \frac{60 \cdot k}{2 \cdot \pi \cdot w} \quad T_{ice}(5000) = 114.615$$



Model Equations for the Driveshaft, ICE, and Planetary Gears

Simulation of a Series Hybrid Electric Vehicle based on Energetic Macroscopic Representation

W. Lhomme¹, A. Bouscayrol¹, Member, P. Barrade²

$$J \cdot \frac{d}{dt} \Omega_{shaft} + f \cdot \Omega_{shaft} = T_{ice} - T_{dcg}$$

$$P_{ice} = m_{ice} \cdot k_{ice} \cdot \Omega_{shaft}$$

$$T_{ice} = m_{ice} \cdot k_{ice} \cdot d_{gas}$$

$$k_{ice} = \frac{\eta \cdot \rho \cdot P_c}{\Omega_{ice_max}}$$

$$\omega_{mg1} + 2.6 \cdot \omega_{mg2} = 3.6 \omega_{ice}$$

J , f , and Ω_{shaft} are the moment of inertia, friction coefficient, and speed of the shaft. T_{ice} and T_{dcg} are the ICE and generator torques. The ICE pressure, P_{ice} , results from the flow of gasoline. η is the ICE efficiency, ρ the density of gasoline, P_c the calorific value of gasoline and Ω_{ice_max} the maximum rotation speed of the engine. The control of the machine is ensured by the m_{ice} ratio, which defines the actual flow of gasoline, d_{gas} , through a specific actuator.

The relation for the shaft rotational velocities ω_{ice} , ω_{mg1} , and ω_{mg2} describes the action of the Hybrid Synergy Drive Planetary Gear (shown in the above illustration).

Fuel_Use := READPRN("http://www.leapcad.com/Transportation/Prius%201.5L%20Atkinson%20Fuel%20Use.TXT")^T

Toyota Prius GIII 1.5L-Vehicle, Motor and Road Parameters:

Max Motor Power:	$P_{max_{m2}} := 50 \cdot \text{kW}$	$P_{max_{m1}} := 25 \cdot \text{kW}$	$P_{max_{ice}} := 57 \cdot \text{kW}$
Max Motor Torque:	$T_{max_{m2}} := 400 \cdot \text{N} \cdot \text{m}$	$T_{max_{m1}} := 207 \cdot \text{N} \cdot \text{m}$	$T_{max_{ice}} := 115 \cdot \text{N} \cdot \text{m}$
Max Motor Speeds:	Max. Acceleration Mode==>	$\Omega_{max_{m1}} := 6500 \cdot \text{RPM}$	$\Omega_{max_{ice}} := 5000 \cdot \text{RPM}$
Gear Ratio and Efficiencies:	$GR := 3.905$	$\eta_{inv} := 0.95$	$\eta_{eng_axle} := 0.95$ $\eta_{axle} := 0.98$ $\eta_{eng_m1} := 0.98$ $r_{tire} := 0.287 \cdot \text{m}$
Motor M2 Constant Power Torque, rpm:	$T_{step_{m2}} := T_{max_{m2}}$	Motor M2 Power Starved rpm	$\Omega_{PS_{m2}} := 400 \cdot \text{RPM}$
Redline := $\Omega_{max_{ice}} \cdot \text{min}$	$\Omega_{CP_{m2}} := \frac{P_{max_{m2}}}{2 \cdot \pi \cdot T_{step_{m2}}}$	Average Cross Wind:	$V_{cw} := 0 \cdot \text{mph}$
Average Wind Velocity:	$V_w := 0 \cdot \text{mph}$	Frontal Area*:	$A_{fg} := 2.33 \cdot \text{m}^2$
Shape Correction Factor:	$SCF := 0.85$	Frontal Area Corrected:	$A_f := A_{fg} \cdot SCF$ $A_f = 1.98 \cdot \text{m}^2$
Drag Coeff:	Cd := 0.26	Rolling Resistance Per Tire: (Average 0% road grade)	$RR_{tire} := 0.008$
Cross Wind Drag Coeff:	$Cd_{cw} := 0.000014$	$\theta := \text{atan}(\theta)$ θ (radians):	
Air Density:	$\rho := 1.293 \cdot \frac{\text{gm}}{\text{liter}}$	Energy _{bat} := 6.5·kW·hr	$P_{max_{bat}} := 21 \cdot \text{kW}$
Road Rolling Resist:	$RR_{road} := 0.007$	Curb Weight:	$M_{curb} := 1317 \cdot \text{kg}$
Rotational Inertia Coeff:	$k_m := 1.08$	Passenger Weight:	$\text{Passengers2} := 170 \cdot \text{lb}$
Gross Weight:	$M_{gross} := M_{curb} + \text{Passengers2}$	$M_{gross} = 3.073 \times 10^3 \text{ lb}$	$M_{batt} := 68 \cdot \text{kg}$

Vehicle Dynamics Equations (We don't know specifics of Prius Control Strategy):

Road Resistance, Fr:	$Fr(v) := M_{gross} \cdot g \cdot [(RR_{tire} + RR_{road}) \cdot \cos(\theta) + \sin(\theta)]$		
Aerodynamic Loss, Fa:	$Fa(v) := 0.5 \cdot \rho \cdot A_f \cdot [(v + V_w)^2 \cdot Cd + Cd_{cw} \cdot (0.5 \cdot v + V_{cw})^2]$	$x := \frac{s}{m}$	
Opposing Force, Fo:	$Fo(v) := Fa(v) + Fr(v)$	$Fo(60 \cdot \text{mph}) = 444.58 \text{ N}$	
Hybrid Synergy Drive, Max. ICE Acceleration:	$\Omega_{ice}(\omega_{mg2}) := \frac{(\Omega_{max_{m1}} \cdot \text{min} + 2.6 \cdot \omega_{mg2})}{3.6}$	$\Omega_{mg2}(\omega_{ice}) := \frac{\omega_{ice} \cdot 3.6 - \Omega_{max_{m1}} \cdot \text{min}}{2.6}$	
Torque, ICE:	$T_{ice}(\omega_{mg2}) := \text{if}[T_{ice}(\Omega_{ice}(\omega_{mg2})) \cdot \text{N} \cdot \text{m} \leq T_{max_{ice}}, T_{ice}(\Omega_{ice}(\omega_{mg2})) \cdot (\text{N} \cdot \text{m}), T_{max_{ice}}]$		
Torque, MG1: $T_{m1}(\omega_{mg2}) := -T_{ice}(\omega_{mg2}) \cdot 3.6^{-1} \cdot \eta_{eng_m1}$	$P_{inv_{m2}}(\omega_{mg2}) := \eta_{inv} \cdot (P_{max_{bat}} - T_{m1}(\omega_{mg2}) \cdot 2 \cdot \pi \cdot \Omega_{max_{m1}})$		
MG2 Inverter Power, P _{INV} :			
Torque, MG2:	$T_{m2}(\omega_{mg2}) := \text{if}\left(\omega_{mg2} \leq \omega_{inv}, T_{max_{m2}}, \frac{P_{inv_{m2}}(\omega_{mg2})}{2 \cdot \pi \cdot \omega_{mg2} \cdot \text{RPM}}\right)$	Solution: $T_{max_{m2}} \Rightarrow P_{inv_{m2}}(\omega_{inv})$	$\omega_{inv} = 926.013$
Tractive Torq, Total:	$T_{tot}(\omega_{mg2}) := T_{m2}(\omega_{mg2}) + T_{ice}(\Omega_{ice}(\omega_{mg2})) \cdot \frac{2.6}{3.6} \cdot \eta_{eng_axle}$	$P_{m2}(\omega) := T_{m2}(\omega) \cdot 2 \cdot \pi \cdot \omega \cdot \text{RPM}$	
Tractive Road Force, Total:	$F_{t\omega}(\omega_{mg2}) := \frac{T_{tot}(\omega_{mg2}) \cdot GR \cdot \eta_{axle}}{r_{tire}}$	$\text{rpm_to_mph}(s) := \frac{GR \cdot s}{(2 \cdot \pi \cdot r_{tire} \cdot \text{RPM})}$	
Tractive Road Force, Total:	$F_{tot}(v) := \frac{T_{tot}(\text{rpm_to_mph}(v)) \cdot GR \cdot \eta_{axle}}{r_{tire}}$	$P_{M2}(\omega) := \frac{P_{m2}(\Omega_{mg2}(\omega))}{\text{kW}}$	
Plot Terms:	$P_{tot}(v) := F_{tot}(v) \cdot v$	$P_{tot}(60 \cdot \text{mph}) = 90.146 \text{ hp}$	$\xi := \frac{1}{(\text{N} \cdot \text{m})}$ $\text{End} := 20$
Third Law of Motion: (dv/dt is acceleration)	Given $\frac{d}{dt}v(t) = \frac{F_{tot}(v(t)) - Fo(v(t))}{k_m \cdot M_{gross}}$	$v(0) = 0$	velocity := Odesolve(t, End)

Applying maximum motor torque, find the velocity starting from initial velocity = 0 mph.

$$\text{accel}(t) := \frac{F_{tot}(\text{velocity}(t)) - Fo(\text{velocity}(t))}{k_m \cdot M_{gross}} \quad P_t(\omega_{m2}) := T_{tot}(\omega_{m2}) \cdot 2 \cdot \pi \cdot \omega_{m2} \cdot \text{RPM} \cdot \text{kW}^{-1}$$

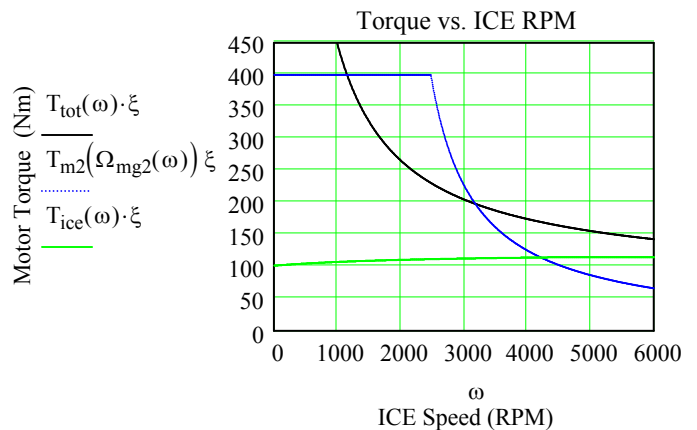
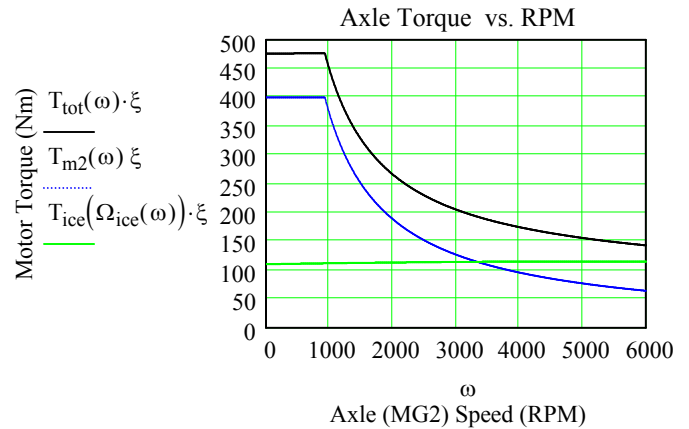
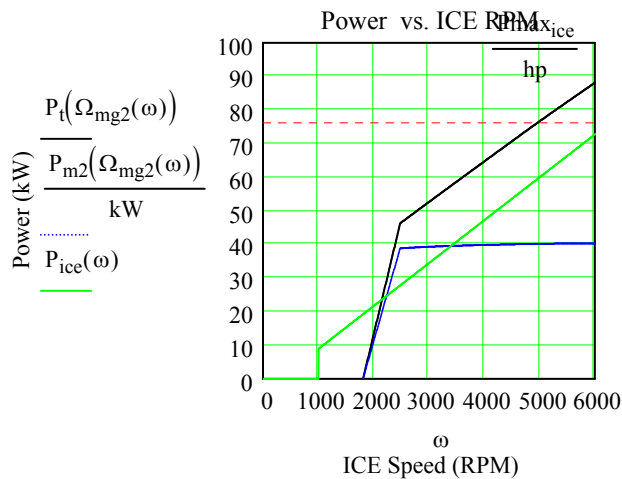
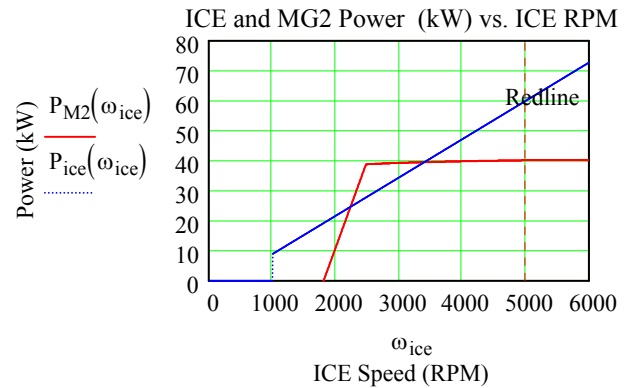
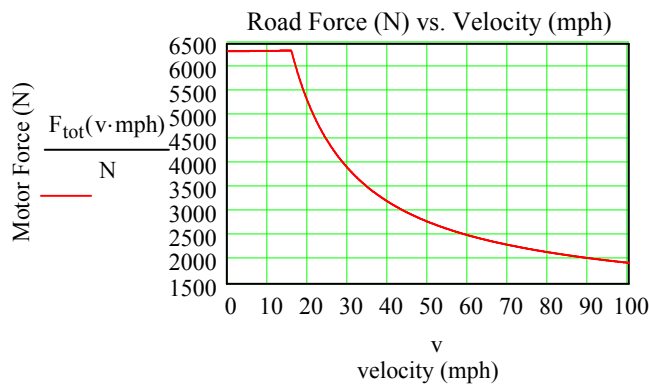
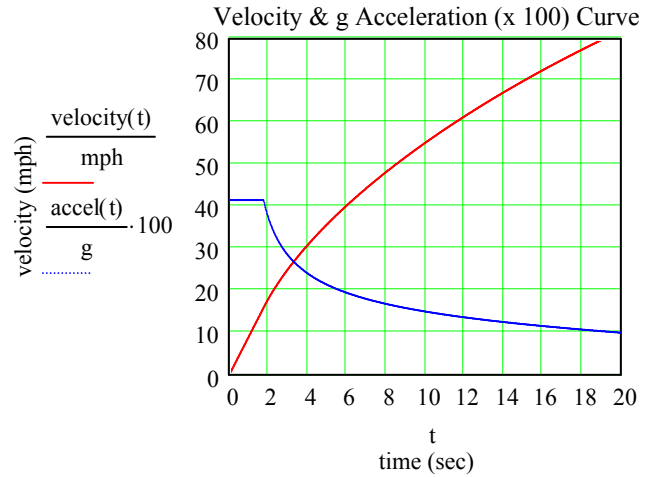
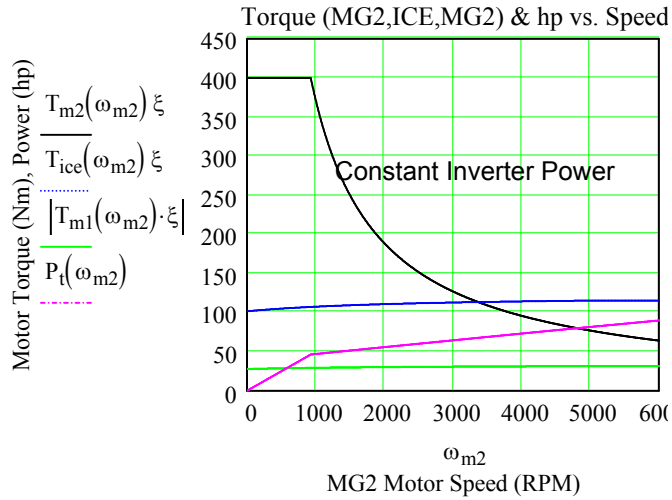
This agrees with Prius Specs

Time := 0·sec

time(v) := root(v - velocity(Time), Time)

time(60·mph) = 11.505 s

PRIUS GEN III PERFORMANCE SIMULATION CURVES



Find the Single Charge (@SOC = 50%) Cruise Range for a given Velocity

Driving Pattern/Profile:

Given we **cruise at constant speed** and Time for start, stop, and regen braking, $Time_{ssr} = \text{every 15 minutes}$.

Drive Train Power Efficiency - Battery Loss to Force Commanded Vehicle Velocity:

State of Charge for generator is SOC_{gen} . **SOC_{gen} is 50% for recharge**. 320V HV battery **idle power is P_o** . 12V battery gives Accessory Power. The Traction Inverter x motor Efficiency - $TInvE$, HV Power Electronics at Idle Efficiency - $IPEE$, and Gear Power Efficiency - GPE are 90%, 95%, and 97%, respectively. Brake Regen efficiency of kinetic energy is 69% @ deceleration = 0.315g. Then the number of starts per hour as a function of velocity, NS , $NumStarts(v, P_o)$, is

$$Time_{ssr} := 30\text{min} \quad TInvE := 0.90 \quad IPEE := 0.95 \quad GPE := 0.97 \quad Regen := 0.69 \quad SOC_{gen} := 0.5$$

USABC Round Trip Battery Energy Efficiency

$$RTEff := 0.92$$

$$Power_{dissLoss}(v, P_o) := \frac{F_o(v) \cdot v}{TInvE \cdot GPE} + \frac{P_o \cdot \text{watt}}{IPEE}$$

$$Energy_{accel}(v) := P_{maxm2} \cdot \text{time}(v)$$

NS_o and NS are iterative converging estimates of $NumStarts$

$$NS_o(v) := 2 \cdot \left(\frac{50 \cdot \text{mph}}{v} \right)^2$$

$$NS(v, P_o, S) := \frac{Energy_{bat} \cdot (1 - S) - NS_o(v) \cdot \left(\frac{Energy_{accel}(v)}{TInvE \cdot GPE} - \frac{Regen \cdot M_{gross} \cdot v^2}{2} \right)}{Power_{dissLoss}(v, P_o) \cdot Time_{ssr}}$$

$$NumStarts(v, P_o, S) := \text{floor} \left[\frac{Energy_{bat} \cdot (1 - S) - NS(v, P_o, S) \cdot \left(\frac{Energy_{accel}(v)}{TInvE \cdot GPE} - \frac{Regen \cdot M_{gross} \cdot v^2}{2} \right)}{Power_{dissLoss}(v, P_o) \cdot Time_{ssr}} \right]$$

$$Cruise_Range(v, P_o, S) := \left[\frac{Energy_{bat} \cdot (1 - S) - NumStarts(v, P_o, S) \cdot \left(\frac{Energy_{accel}(v)}{TInvE \cdot GPE} - \frac{Regen \cdot M_{gross} \cdot v^2}{2} \right)}{Power_{dissLoss}(v, P_o)} \right] \cdot v$$

Single Charge Highway Cruise Range Estimate

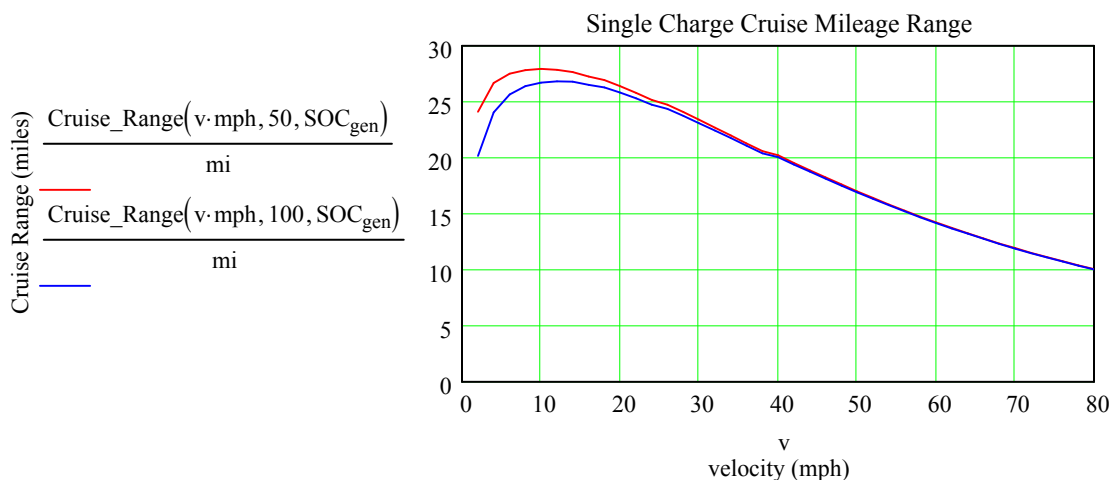
- $Cruise_Range(30\text{-mph}, 50, SOC_{gen}) = 23.38 \text{ mi}$
- $Cruise_Range(40\text{-mph}, 50, SOC_{gen}) = 20.207 \text{ mi}$
- $Cruise_Range(50\text{-mph}, 50, SOC_{gen}) = 16.995 \text{ mi}$
- $Cruise_Range(55\text{-mph}, 50, SOC_{gen}) = 15.548 \text{ mi}$
- $Cruise_Range(60\text{-mph}, 50, SOC_{gen}) = 14.221 \text{ mi}$
- $Cruise_Range(62.5\text{-mph}, 50, SOC_{gen}) = 13.602 \text{ mi}$
- $Cruise_Range(70\text{-mph}, 50, 0.5) = 11.918 \text{ mi}$

Velocity Range

$v := 0, 2.. 80$

$SOC_{gen1} := 0.42$

$Cruise_Range(70\text{-mph}, 50, 0.5) = 11.918 \text{ mi}$



Cruise Range as a Function of Traction Battery Idle Power, P_0

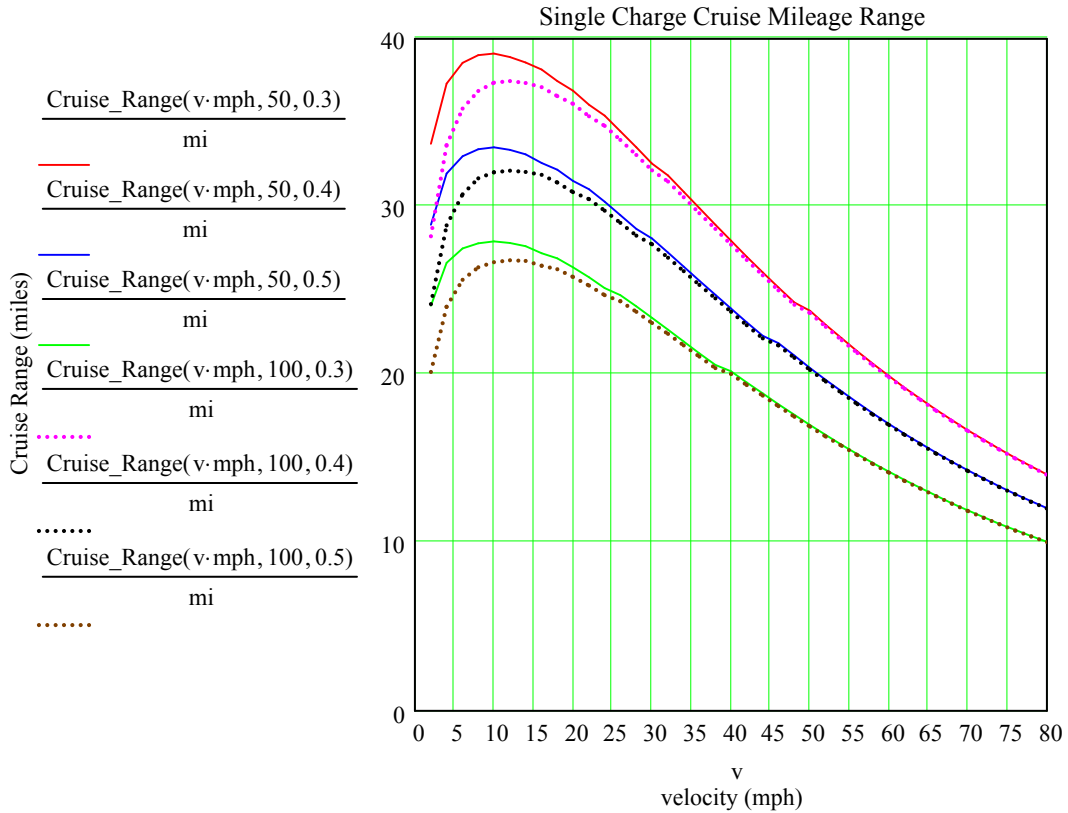
$$\text{Cruise_Range}(15\text{-mph}, 50, 0.3) = 38.294 \text{ mi}$$

$$\text{Cruise_Range}(55\text{-mph}, 50, 0.5) = 15.548 \text{ mi}$$

$$180 \cdot \frac{\text{km}}{\text{hr}} = 111.847 \frac{\text{mi}}{\text{hr}}$$

$$\frac{0.5 - 0.25}{0.5} = 0.5$$

$$\frac{\text{Cruise_Range}(55\text{-mph}, 100, 0.25) - \text{Cruise_Range}(55\text{-mph}, 100, 0.5)}{\text{Cruise_Range}(55\text{-mph}, 100, 0.5)} = 0.5$$



$k := 1000$

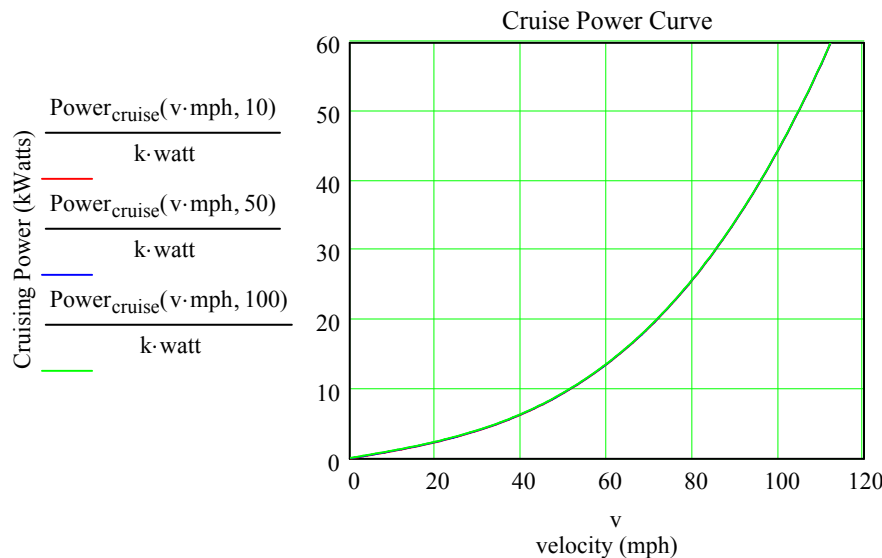
Find the Power to Maintain Constant Velocity

Note: The generator's output is 54 kW. This allows it produce a net charge up to 80 mph cruise.

$$\text{Power}_{\text{cruise}}(v, P_0) := \text{Power}_{\text{dissLoss}}(v, P_0)$$

$$\text{Power}_{\text{cruise}}(60 \cdot \text{mph}, 100) = 13.765 \text{ kW}$$

$$v := 0, 1 \dots 120$$



```
FTPF := READPRN("http://www.leapcad.com/Transportation/FedTestProc.TXT" )
```

```
UDDSF := READPRN("http://www.leapcad.com/Transportation/uddscol.txt" )
```

```
HWYF := READPRN("http://www.leapcad.com/Transportation/hwycol.txt" )
```

```
FP10 := READPRN("http://www.leapcad.com/Transportation/FTP10Hz.TXT" )
```

```
HY10 := READPRN("http://www.leapcad.com/Transportation/HWY10Hz.txt" )
```

```
US06F := READPRN("http://www.leapcad.com/Transportation/US06PROFILE.TXT" )
```

AER Given Three Different Driving Schedules

Read US06 and FTP Driving Profile Files

<http://www.epa.gov/nvfel/testing/dynamometer.htm>

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds.

The Federal Test Procedure(FTP) is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. FP10 is a 10 Hz Sampling. HY10 is the 10 Hz Highway schedule.

$$t := \text{FTP}^{(0)} \quad \text{FTP} := \text{FTP}^{(1)} \quad \text{rows}(\text{FTP}) = 1.875 \times 10^3$$

$$\text{UDDS} := \text{UDDSF}^{(1)} \quad \text{rows}(\text{UDDS}) = 1.37 \times 10^3$$

$$\text{HWY} := \text{HWYF}^{(1)} \quad R_{\text{hwy}} := \text{rows}(\text{HWY})$$

$$\text{FTP10V} := \text{submatrix}(\text{FP10}, 0, \text{rows}(\text{FP10}) - 1, 1, \text{cols}(\text{FP10}) - 1)$$

$$\text{HWY10V} := \text{submatrix}(\text{HY10}, 0, \text{rows}(\text{HY10}) - 1, 1, \text{cols}(\text{HY10}) - 1)$$

$$\text{time} := \text{US06F}^{(0)} \quad \text{US06} := \text{US06F}^{(1)} \quad n_6 := 0 \dots 598$$

All Electric Range, AER, for Driving Profile Velocity/Time File, P Sampling Rate, Hz, and SOC = 0

Regen Efficiency Curve vs Decel (g): $REff(g) := \frac{85}{77} \cdot 0.01 \cdot \left[\left(1 - e^{-27.129 \cdot g} \right) \cdot 91.235 - 28.408 \right]$ $Gg := \frac{\text{mph}}{\text{sec} \cdot g}$

```

AER(P, Hz) :=
  Ebat ← E_diss ← v_old ← 0
  n ← -1
  N ← rows(P) - 1
  while E_diss <  $\frac{\text{Energy}_{\text{bat}}}{\text{kW} \cdot \text{hr}}$ 
    n ← n + 1
    t ← mod(n, N)
    v ← P_t
    v_avg ← (v + v_old) · 0.5
    P_accel ←  $\frac{k_m \cdot M_{\text{gross}} \cdot (v - v_{\text{old}}) \cdot \frac{\text{mph} \cdot \text{Hz}}{\text{sec}} \cdot v_{\text{avg}} \text{mph}}{T_{\text{InvE}} \cdot GPE}$  if v > v_old
    P_accel ←  $k_m \cdot M_{\text{gross}} \cdot (v - v_{\text{old}}) \cdot \frac{\text{mph} \cdot \text{Hz}}{\text{sec}} \cdot v_{\text{avg}} \text{mph} \cdot REff[(v_{\text{old}} - v) \cdot \text{Hz} \cdot Gg]$  otherwise
    E_diss ← E_diss +  $\frac{(\text{Power}_{\text{dissLoss}}(v \cdot \text{mph}, 100) + P_{\text{accel}}) \cdot \text{sec}}{\text{kW} \cdot \text{hr} \cdot \text{Hz}}$ 
    v_old ← v
    Ebat_n ← E_diss
  R ←  $\sum_{m=0}^n \frac{(P_{\text{mod}(m, N)} + P_{\text{mod}(m+1, N)}) \cdot \text{mph} \cdot \text{sec}}{2 \cdot \text{mi} \cdot \text{Hz}}$ 
  R
  
```

$r1 := 0 \dots \text{rows}(\text{HWY10}) \cdot 10 - 1$ $\text{HWY10}_{r1} := \text{HWY10V}_{\text{ceil}\left(\frac{r1+1}{10}\right)-1, \text{mod}(r1, 10)}$

$\text{AER}(\text{US06}, 1) = \blacksquare$ $\text{AER}(\text{FTP}, 1) = 28.232$ $\text{AER}(\text{HWY}, 1) = 27.23$ $\text{AER}(\text{HWY10}, 10) = \blacksquare$

EPA 20085 Cycle MPG Fuel Economy Least Squares Fit Regression for AER to SOC = 0

$\text{MPG}_{\text{city}} := \frac{1}{\left(0.003259 + \frac{1.18053}{\text{AER}(\text{FTP}, 1)} \right)}$ $\text{MPG}_{\text{city}} = 22.186$ $\text{MPG}_{\text{hwy}} := \frac{1}{0.001376 + \frac{1.3466}{\text{AER}(\text{HWY}, 1)}}$ $X := \frac{1}{40}$

$\text{MPG}_{\text{epa}} := 0.55 \cdot \text{MPG}_{\text{city}} + 0.45 \cdot \text{MPG}_{\text{hwy}}$ $\text{MPG}_{\text{epa}} = 21.055$

$r := 0 \dots \text{rows}(\text{FTP}) - 1$ $\text{Distance}_r := \sum_{r=0}^r \text{FTP}_r \cdot \frac{10}{60 \cdot 60}$ $\text{rr} := 0 \dots \text{rows}(\text{US06}) - 1$ $\text{Distance}_{\text{rr}} := \sum_{\text{rr}=0}^{\text{rr}} \text{US06}_{\text{rr}} \cdot \frac{10}{60 \cdot 60}$
 $\text{max}(\text{Distance}) = 110.414$ $\text{max}(\text{Distance}) = 80.08$

WRITEPRN("EFTP.PRN") := AER(FTP, 1)·40 $E_{FTP} := \text{READPRN("EFTP.PRN")}$ $\max(E_{FTP}) \cdot X = 28.4$
 WRITEPRN("EUS06.PRN") := AER(US06, 1)·40 $E_{US06} := \text{READPRN("EUS06.PRN")}$ $\max(E_{US06}) \cdot X = 19.66$
 WRITEPRN("EHWY.PRN") := AER(HWY, 1)·40 $E_{HWY} := \text{READPRN("EHWY.PRN")}$ $\max(E_{HWY}) \cdot X = 27.4$

