Tesla S P100D Ludicrous Plus Mode Simulation

2-22-2017

The Simulation can be run or modified with Mathcad 14/15. Free Trial at**: http://www.ptc.com/product/mathcad/free-trial**

Mathcad Simulation at: http://www.LeapCad.com/TeslaSP85DLudicrousPlusModeSimulation.xmcd

Goal: Simulate Ludicrous + Mode Dynamic Power Train Performance

This paper shows a macro model for performance simulation of a Tesla S P100D Ludicrous **Plus** Mode. Results were calculated for 100% and 50% State of Charge (SOC). The key parameters are peak motor torque, peak battery power (SOC), curb weight, maximum tire traction, and some assumptions about the power loss/efficiency high efficiency induction motors (93%), and Inverter and power train (87%). Net System Efficiency, SysEff ~ **81'** For **50% SOC**, battery max power is 760 hp, System **Power of 760 hp**. From statements of Elon Musk, we wou infer that tire traction is capable of gripping the road @1.4g. Assume that with sufficient power, we can get a pea combined (front and rear motor) Motor Torque of **791 ft lb***. Under these assumptions, the model shows that we meet the 0 to 60 mph in < 2.5 sec Spec for 50% SOC. This Analysis in done in the following ten Sections. Sectic IV considers four different traction scenarios (Sec IV, pg. 4).
The Calculations & graphs in **Sections III to VI @ 100% SOC.**

Note: Drag Times Dyno tests gave a max torque of **864 ft lb** See:
http://www.greencarreports.com/news/1098611_too-much-electric-car-torque-tesla-model-s-p85d-breaks-dynamometer-video

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APPENDIX

I. Introduction - Simple Analysis

Examining the Difference Between Insane & P100D Ludicrous Plus Speed Upgrade

Insane Mode Specifications
3.1 seconds 0-60 mph
Peak Acceleration: 1 g
Front and Back Motor Spec: Power **691 hp**
Conventional Fuse: 1300 amp battery limitation
Battery Peak Power: 1300 A x 398.6 = **695 hp**
Inverter and/or Algorithm sets max accel ~ 1g.

Ludicrous Mode Plus Specifications
2.5 seconds 0-60 mph with Ludicrous Speed Upgrade. 1/4 mile in 10.7 s
Top speed 217 mph @ Max Motor RPM ~18,000
Front and Back Motor Power **Spec**: **768 hp , 573 kW**
Electronic Fuse (P85D): 1500 amp "effective" battery upgrade
Battery 8256 cells @50% SOC Peak Power: 1762 A x 314V = **741 hp**
Peak Motor Torque: 864 ft lb. (See Drag Times P85D Dyno Data)
New Inverter Algorithm to maximize acceleration. Motor Trend 1.4 g

We assume that the implementation of the **New Algorithm** for Rear Motor Torque & Power Split in Ludicrous Mode, will provide the maximum tire grip acceleration, 1.4g, and not optimize Efficiency, that is, minimize Power: = TS_{rear} = 503/809 = 0.63

What is the minimum acceleration needed to meet the 0 to 60 mph in 2.5 seconds? There are two factors involved in this specification, velocity, v, and time, t. For the sake of this *Simple Analysis*, let's assume that the acceleration is constant. Now acceleration defined as the rate of change of velocity. In symbols, the constant acceleration, $a._{constant}$ = change of velocity/time. Then the acceleration needed to get to 60 mph in 2.5 seconds, or 60 mph/2.5 s, is a constant 24 mph per sec.

Thus a **constant** 24 mph/s in sufficient to meet our spec. In units of the earth's gravitation g, 1 g = 21.9 mph/s. We need a constant/peak of ~1.2/1.4 g. (Motor Trend data g_{max}=1.4g) Then to accelerate a mass of 5,051 lb, we need a Force of 6612/7071 lb or for 14" tire radius and 9.73 Gear Ratio, a Motor Torque of 726/848 ft lbf. Then 864 ft lbf should be more than sufficient.

However, torque or acceleration is not constant. The 864 ft lb is peak torque. Torque falls off with vehicle speed or if the battery power is not sufficient to supply enough power to keep it at its peak. Also, Ludicrous Plus Mode uses a New Algorithm that provides complex Traction Control for max acceleration. Thus a more in-depth analysis is required. Among other things, it requires working out what the torque falloff is with speed. We will find that with a net combined Inverter and Motor Power Efficiency of 81% and high performance 1.4 g tires (implied by Elon Musk), we can meet the Ludicrous Specs.
This more in-depth analysis is what follows.

II. Macro Performance Model Discussion & Description of the Model

Macro Model: Macro Models requires only limited knowledge of internal parameters. We treat the system as a Black Box. That is, we don't know the details of what's inside, just a few fundamental parameters. We are only interested in overall performance. Ignore the intricacies. Simple, but not too simple. May not know what is inside, but regardless, the laws of Physics still apply. We just need basic physical parameters such as:

Vehicle mass (Mcurb), Coefficient of tire friction μ, and radius, Gear Ratio GR, max motor Torque & Power, battery power, and System Power Efficiency (Inverter, Gears, and Motors).
The vehicle also has rotational energy from rotating tires, motor rotor, and gear box.
A factor km, which multiplies the mass, accounts for this added rotational mass.
$M_{curb\,=}$ 4936 lbm, μ = 1.4 (equivalently, max g = 1.4), 265/35ZR21 tires, tire radius=14 inches, GR = 9.73.

$r_{tire} := 13.5in$

$GR := 9.73$

$M_{curb} := 4891lbm$

$k_m := 1.0447$

Then acceleration (a) is given by:

Newton's Second Law: a = Traction Force/m = Torque x GR/Mcurb

See pg 3 for Section on **Traction Control**.

Then the Torque required to get to g = 1.4, requires that Torque be at least:

Torque_max_g = Weight x km x 1.4 g tire radius/GR = **857 ft lbf**

$$T_{max\_g} := M_{curb} \cdot k_m \cdot \frac{1.4g \cdot r_{tire}}{GR}$$

$$T_{max\_g} = 827.1 \cdot ft \cdot lbf$$

The present max Torque spec is 791 **ft lbf.** Drag Times Dyno testing showed that the max 864 ft lb is possible.
Where does the increaed torque come from? It comes from increased battery current.
The peak Torque is proportional to the peak current.

$T_{max} := 791 ft \cdot lbf$

$Power_{max} := 760hp$

What is the Estimated Motor Power needed to meet the 0 - 60 mph in 2.5 s performance, P_{Spec}?

Ludicrous Mode: The current has been increased from 1300 to 1500 amps (15 %), the total dual motor power increased from 682 hp to **762 hp** (12%), and the total torque from 707 to 713 ft lbf. However, as we saw in section I, the System Power to the motor @81% System Efficiency is limited to **649 hp**

There is a basic relationship between Torque, Motor RPM, and Motor Power:
Assume that there is No Traction Control, this is tires can slip.
Thus initially, full torque is applied to the wheels, until max motor power limits the torque.
Refer to the below plot and examine the Power versus time profile.
The Power is given by: Power = Torque x Angular Velocity, until Max Power is reached.
This is shown in the graph below. Tire velocity, v_{Pmax}, to get to max motor power and Torque =
The time to get to max motor power is t_{Pmax}. The velocity at which this occurs is v_{Pmax}.
There are 2 paths to get to the max power: #1 Tires allowed to slip and #2 Tires do not slip (Pg. 3).
Elon Musk said that the peak acceleration is 1.4 g. Designate the time to reach full power, t_{Pmax}.

Velocity at Max Power

$$v_{Pmax} := \frac{Power_{max} \cdot r_{tire}}{T_{max} \cdot GR}$$

$v_{Pmax} = 41.66 \cdot mph$

$t_{Pmax} := 1.46s$

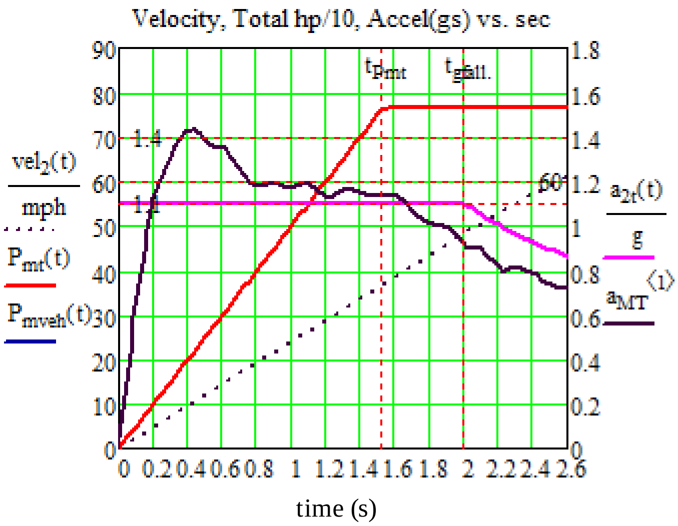
$$Pm(\omega) = T(\omega) \cdot k \cdot 2 \cdot \pi \cdot \omega \text{ RPM}$$

The average power from the start at 0 power to the peak of $Power_{max}$ is 1/2 $Power_{max}$.
If the time to get motor Power max is t_{Pmt}, then the Energy is 1/2 $Power_{max}$ x t_{Pmax}.
This energy is equal to the Kinetic Energy in going from zero velocity to velocity, v.
The relationship; is shown at the right.

The 2 motor power curves belong to two different Traction Models:
#1: No Traction Control, Red Curve
#2: With Traction Control, Blue Curve

t_{Pmt} is the time to Peak Power = 1.5s
t_{gfall} is the time for acceleration(g) to fall below 1.1g = 2 s.
These times are in seconds.

The most critical part of performance is the 1.5 seconds of Peak Power from the time the acceleration falls from 1.4 g to the 2.5 second spec limit.
The black curve is Motor Trend's acceleration (gs) test data (**test allows 1 foot of roll out =0.26 s).**
Note: Power, P (hp) curves at left, are divided by 10 to allow the same scale as the velocity in mph.



$M_{gross} := M_{curb} + 160lbm$

$$KE(v) := \frac{1}{2} \cdot M_{gross} \cdot k_m \cdot (v)^2$$

$KE(60mph) = 0.24 \cdot kW \cdot hr$

Note that the velocity vs. time curve is almost a straight line. It **approximately equivalent** in performance to a constant acceleration of **1.1 g** during the 2.5 seconds to get to 60 mph.
The Areas of a_2 & a_{MT} vs time must be greater than 1.1g x 2.5s rectangle to get to 60 mph in 2.5s.

What Max Power do we need to meet the 0 - 60 mph in 2.5 s with No Traction Control?
For a vehicle velocity, v, the the Kinetic Energy, KE, to get to v is given by:

$$E_{motor} := \frac{1}{2} \cdot 760hp \cdot 1.5s + 760hp \cdot (2.5 - 1.5)sec = 0.28 \cdot kW \cdot hr$$

This demonstrates that 760 hp is sufficient to give enought energy to accelerate 0 - 60 mph in 2.5 sec. What follows is a more detailed analysis.

III. Specifications & Engineering Estimates: Ludicrous Mode Peak Acceleration

System Efficiency: $SysEff := 0.90$ % SOC Voltages: $V_{batt\_100} := 394volt$ $V_{batt\_80} := 384volt$ $V_{batt\_50} := 360volt$ $V_{batt} := V_{batt\_50}$

Results are Shown for **100% State of Charge**.

Battery and System Power **@100% SOC** $Power_{Batt} := V_{batt} \cdot 1762A = 850.64 \cdot hp$ $Power_{System} := Power_{Batt} \cdot SysEff = 765.57 \cdot hp$

85 kW-hr Battery

De-acceleration Battery Energy Regeneration Factor: $Regen := 0.64$

New Algorithm for Rear to Front Motor Torque & Power Split to maintain **peak acceleration**: = T_{Split} = 503/259 ~ 1.94 Gear Ratio:

Ludicrous + Mode Max Power: $Power_{max} := 741 \cdot hp$ $RPM_{max} := 18000$ $T_{Split} := 1.94$ $GR := 9.73$

Limit: $Power_{System} = 765.57 \cdot hp$ $Power_{max} := Power_{System}$ Battery Energy: $Energy_{bat} := 100 \cdot kW \cdot hr$ $R_{phase} := 0.006ohm$

Max Dual Motor Torque: $Power_{max} = 765.57 \cdot hp$

From TeslaMotors.com/models

$Torque_{maxOld} := 707 \cdot ft \cdot lbf$

245/35R19 Tire Radius: $r_{tire} := \frac{27.75}{2} in = 13.88 in$

Ultra High Performance Tire

Drag Times Dyno Testing $T_{DTmax} := 858 \cdot ft \cdot lbf$

Torque SAE Net $T_{max} := 791 \cdot ft \cdot lbf$ $F_{Motor\_Max} := \frac{T_{max} \cdot GR}{r_{tire}}$

Tire Coefficient of Friction, μ: $\mu := 1.1$ $car_{max\_g} := \mu \cdot g = 1.1 \cdot g$ $k := 1000$ $\tau := 1 \cdot sec$

Max 1.4 g Quoted by Elon Musk

Curb Weight: $M_{curb} := 4891lbm$ $M_{gross} := M_{curb} + 160lbm = 5051 \cdot lbm$

Aerodynamic Drag Coeff (TM): $Cd := 0.22$ Average Wind Velocity: $Vw := 0 \cdot mph$ $g_{max} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire} \cdot g}$

Cross Wind Drag Coff: $Cd_{cw} := 0.000014$ Effective Cross Wind V: $V_{cw} := 0 \cdot mph$

Shape Correction Factor: $SCF := 0.85$ Vehicle Frontal Dimensions: $Af := (57 - 7.9)in \cdot 77 \cdot in$ $g_{max} = 1.26$

Air Density, tire resistance: $\rho := 1.293 \cdot \frac{gm}{liter}$ Drag Frontal Area $Ad := Af \cdot SCF$ $Ad = 2.07 \cdot m^2$

Road Rolling Resistance: $RR_{road} := 0.007$ Tire Rolling Resist, Hys: $RR_{tire} := 0.011$ $T_{hys} := 0 \cdot \frac{sec}{m}$

Effective Mass Coefficient: $k_m := 1.0447$ **EPA Range Spec** for P100 is 315 miles See page 6 for $EPA\_RangeSim := 251mile$

IV. Tire Traction & Control Models: #1 Perfect Grip, #2 Tires slip, #3 No Slip, #4

Simple Step Model of Tire Traction (Assume perfect weight distribution per motor, i.e. same acceleration at each motor)
Depending on road conditions, Tires do not have perfect grip, they may slip. Vehicle acceleration, a_{veh} is limited to the maximum tire traction ($tire_{max\_g}$) = 1.4g. The tire rpm x GR = motor rpm, but because of slip, tire velocity can be greater than vehicle velocity. Therefore, vehicle acceleration and velocity are not directly proportional to rpm, that is, tires may slip:

Case #1 $v_{tire} = v_{vehicle}$
No Traction Control, but no tire slip.
Max motor power and torque are applied to tires. Perfect Tires that do not slip. Acceleration can **exceed 1.4 g**.

Case #2 $v_{tire} = v_{vehicle}$
Perfect Traction System and High Performance g = 1.4 tires.
Because of Traction Control and tire slip, effective motor rpm can be greater than vehicle speed during tire slip or Traction Control. Vehicle speed depends on **tire** coefficient of resistance, μ, which is equal to 1.4 for Ludicrous+. This allows a **max of 1.4 g**. For Case #2, we assume Traction Control limits g to 1.4.

Macro Model of Motor Dynamics: Velocity of Tire is v

Angular Velocity Symbol, Ω (units of radians/second) $\Omega(\omega) := 2\pi 1000\cdot\omega\cdot min^{-1}$ RPM/1000 Symbol, ω_k $RPM := min^{-1}$

Angular Vel Ω @Max Power: $\Omega_{Pmax} := Power_{max}\cdot T_{max}^{-1}$ $RPM_{Pmax} := \frac{\Omega_{Pmax}}{2\cdot\pi}$ $RPM_{Pmax} = 5083.28\cdot RPM$

Convert velocity to RPM $VtoRPM(v_v) := v_v\cdot(1000\cdot 2\cdot\pi\cdot r_{tire}\cdot RPM)^{-1}$ $\omega_{Pfall} := RPM_{Pmax}\cdot k^{-1} = 5.08\cdot RPM$

Tire Velocity at Torque Fall: $v_{Tfall} := RPM_{Pmax}\cdot 2\cdot\pi\cdot r_{tire}\cdot GR^{-1}$ $v_{Tfall} = 43.13\cdot mph$

Tire Velocity to kRPM: $VtokR(v_t) := v_t\cdot(k\cdot 2\cdot\pi\cdot r_{tire}\cdot RPM)^{-1}$ $VtokR(60\cdot mph)\cdot GR = 7.07$

Road Resistance, Ft: $Ft(v_v) := M_{gross}\cdot g\cdot\left[T_{hys}\cdot v_v\cdot\sin(\theta) + (RR_{tire} + RR_{road})\cdot\cos(\theta) + \sin(\theta)\right]$ RPM_{pmax} for Max Power:

Air Drag Force, Fa: $Fa(v_v) := 0.5\cdot\rho\cdot Ad\cdot\left[(v_v + Vw)^2\cdot Cd + Cd_{cw}\cdot(V_{cw})^2\right]$

Note: For Drag and Road Resistance, approximate vehicle with v_{tire}. At <60mph Compared to Ftire, Fo is small.

Total Opposing Force, Fo: $Fo(v_v) := Fa(v_v) + Ft(v_v)$ $Fo(60\cdot mph) = 138.61\cdot lbf$

Torque/Force Falloff Curve: $\omega_{kmax} := 15.8\cdot RPM$ $T_{PLt}(\omega_k) := Power_{max}\cdot\Omega(\omega_k)^{-1}$ $T_{PLt}(55) = 73.11\cdot ft\cdot lbf$

Tm is Torque of motor
Fmot, Tractive Force from motor, not from slipping tires:

$T_m(\omega_k) := if(\omega_k\cdot RPM \geq \omega_{Pfall}, T_{PLt}(\omega_k), T_{max})$ $P_m(\omega_k) := T_m(\omega_k)\cdot k\cdot 2\cdot\pi\cdot\omega_k\cdot RPM$

$T_{mv}(v_t) := T_m(VtokR(v_t)\cdot GR)$ $F_{mot}(v_t) := \frac{GR}{r_{tire}}\cdot T_{mv}(v_t)$ $F_{PL}(v_t) := Power_{max}\cdot(v_t\cdot mph)^{-1}$

Solve for Velocity, Acceleration, and Distance versus Time

We are using Mathcad 14, a Computer Math Program, to do the Calculations: http://www.ptc.com/product/mathcad/free-trial

Case 1: Perfect Grip Tires at Maximum Motor Power, Coefficient of Tire Friction > 1.4 g

Newton's Third Law of Motion: $T_{max} = 791\, ft\cdot lbf$

$$a_1(v) := \frac{F_{mot}(v) - Fo(v)}{k_m\cdot M_{gross}}$$ $$a_{1Tmax} := \frac{T_{max}\cdot GR}{M_{gross}\cdot k_m\cdot r_{tire}} = 1.26\cdot g$$

$V := 0\cdot mph$ $$vel_1(t) := root\left(t\cdot sec - \int_0^V \frac{mph}{a_1(V\cdot mph)}\,dV, V\right)\cdot mph$$ $$time_{a1}(v) := \int_0^v \frac{1}{a_1(v)}\,dv$$

$time_{a1}(60mph) = 2.33\,s$
$vel_1(2.22) = 57.87\cdot mph$

$v_{gfall} := root(a_1(V\cdot mph) - car_{max\_g}, V) = 48.44$ $time_{a1}(60mph) = 2.33\,s$ $a_{1t}(t) := a_1(vel_1(t))$

Velocity g fall, a ≤ 1.4g $a_1(v_{gfall}\, mph) = 1.1\cdot g$ $a_{1t}(0) = 1.24\cdot g$

Case 2: High Performance 1.4 g Tires and Acceleration Limited to 1.1 g, but Use Max Power and No Spin

a_2 acceleration is allowed by high performance tires on dry road.

$car_{max\_g} = 1.1 \cdot g$

$a_2(v) := if(a_1(v) \geq car_{max\_g}, car_{max\_g}, a_1(v))$

$vel_2(t) := root\left(t \cdot sec - \int_0^V \frac{mph}{a_2(V \cdot mph)} dV, V\right) \cdot mph$ $\quad time_{a2}(v) := \int_0^v \frac{1}{a_2(v)} dv$ $\quad vel_2(2.55) = 60.08 \cdot mph$

$a_2(v_{gfall}\, mph) = 1.1 \cdot g$

a as function of time, not velocity $\quad a_{2t}(t) := a_2(vel_2(t))$ $\quad t_{gfall} := time_{a2}(v_{gfall} \cdot mph) = 2.01\, s$ $\quad a_{2t}(0mph) = 1.1 \cdot g$ $\quad time_{a2}(60mph) = 2.55\, s$

$vel_2(11.1) = 147.7 \cdot mph$ $\quad distance_2(10.1) = 0.25 \cdot mile$

RPM at g fall: $R_{gfall} := VtokR(v_{gfall} \cdot mph) \cdot GR$

Case 3: Traction Control - Tire Force is Power Limited - No Tire Spin (This model not yet perfected)

$F_{1\_1g} := k_m \cdot M_{gross} \cdot car_{max\_g} = 5804.46 \cdot lbf$ $\quad T_{1\_1g} := \frac{F_{1\_1g} \cdot r_{tire}}{GR}$ $\quad T_3(\omega_k) := if\left(\omega_k \leq R_{gfall}, T_{1\_1g}, Power_{max} \cdot \Omega(\omega_k)^{-1}\right)$

$P_{1\_1g}(\omega_k) := T_{1\_1g} \cdot \omega_k \cdot k \cdot 2 \cdot \pi \cdot RPM$ $\quad P_{1\_1g}(5.252) = 689.75 \cdot hp$ $\quad \omega_{3Pmax} := 5252$

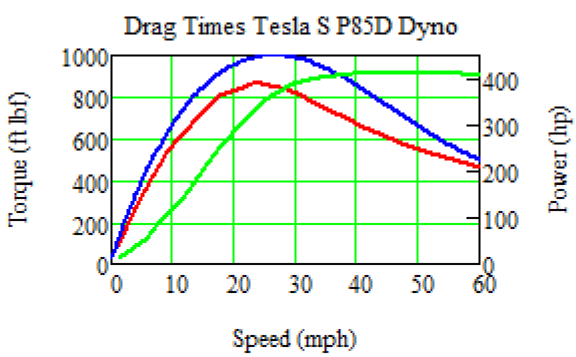
$P_3(\omega_k) := T_3(\omega_k) \cdot (\omega_k \cdot k \cdot 2 \cdot \pi \cdot RPM)$ $\quad F_3(v) := \frac{GR}{r_{tire}} \cdot T_3(VtokR(v \cdot mph) \cdot GR)$

Case 3: We end up getting the same effective peak torque, we just don't waste the power put into spinning wheels. Tesla has a patent on how to split the power

Case 4: Fit Torque to Drag-Times Dyno Torque Curve -->Even @100% Efficiency, Does Not Meet Specs, aft

This Model based on a Torque Shape Function is not yet fully developed. It does not give correct values.

$a_4(v)$ = P85D Acceleration/Torque, $a_1(v)$ x $T_{Shape}(v)$:
$T_{Shape}(v)$ was fitted to have the same shape, as the **Drag Times Torque/speed Curve - See Graph Below**



Dyno Data by Drag Times for 2015 Tesla S P85D
Dyno Torque (Red) and Dyno Power (Green)
DynoTorque Shape **x 1000** (Blue), Extracted from Dyno Torque
http://www.dragtimes.com/2015-Tesla-Model-S-Videos-27143.html

$car_{max\_g} := 1.4 \cdot g$ $\quad$ a = Torque*(GR/r_{tire})/mass

$a_{DT}(v) := if\left(a_1(v) \cdot T_{Shape}\left(\frac{v}{mph}\right) \geq car_{max\_g}, car_{max\_g}, a_1(v) \cdot T_{Shape}\left(\frac{v}{mph}\right)\right)$

$vel_4(t) := root\left(t \cdot sec - \int_0^V \frac{mph}{a_{DT}(V \cdot mph)} dV, V\right) \cdot mph$ $\quad vel_{4.}(t) := \frac{vel_4(t)}{mph}$

$vel_4(2.) = 22.47\, mph$

$a_{DTt}(t) := a_{DT}(vel_2(t))$ $\quad time_{at}(v) := \int_0^v \frac{1}{a_{DT}(v)} dv$

Does not meet Specs

$time_{DT}(40mph) = ▮$

Case 5: Extract Acceleration from Motor Trend P100D Road Test Data. Predict Velocity(t) and Torque(t).

Motor Trend (MT) did some testing and got acceleration curves for three of the quickest production cars

Article: 2017 TESLA MODEL S P100D FIRST TEST: A NEW RECORD — 0-60 MPH IN 2.28 SECONDS!
http://www.motortrend.com/cars/tesla/model-s/2017/2017-tesla-model-s-p100d-first-test-review/

I abstracted (got data points) from the Tesla P100D accel plot and put data points into a table: a_{MT}

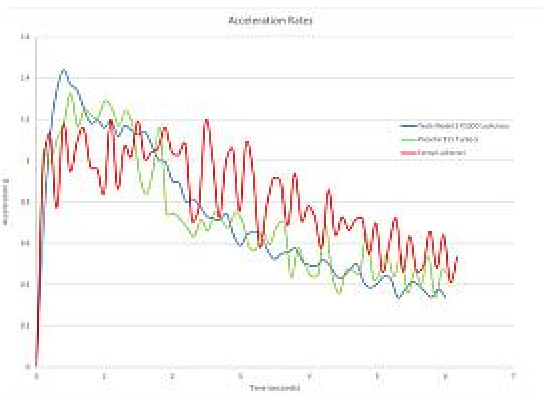
$a_{MTx} := \text{READPRN("Tesla P100D accel Table2.txt")}$ $\quad a_{MT} := \text{submatrix}(a_{MTx}, 0, 83, 0, 1)$ $\quad u := 0..83$ $\quad t_{MT_u} := a_{MT_{u,0}}$

$\text{length}\left(a_{MT}^{\langle 0\rangle}\right) = 84$ $\quad uu := 0..77$ $\quad 1\ \text{mph} = 1.467\ \text{ft/s}$ $\quad u2 := 0..76$

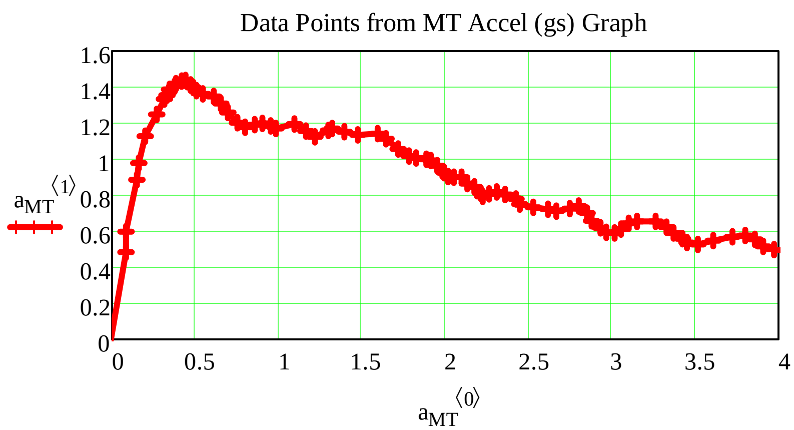
I calculated velocity and distance from the abstracted acceleration versus time data points.

$$v_{MT_{uu}} := \sum_{u=0}^{uu}\left[\frac{1}{2}\left(a_{MT_{u,1}} + a_{MT_{u+1,1}}\right)\cdot 22\cdot\left(a_{MT_{u+1,0}} - a_{MT_{u,0}}\right)\right] \qquad d_{MT_{u2}} := \sum_{u2=0}^{u2}\left[\frac{1.47}{2}\left(v_{MT_{u2}} + v_{MT_{u2+1}}\right)\cdot\left(t_{MT_{u2+1}} - t_{MT_{u2}}\right)\right]$$

MT data. Blue curve is Tesla. See Section X and XI below.

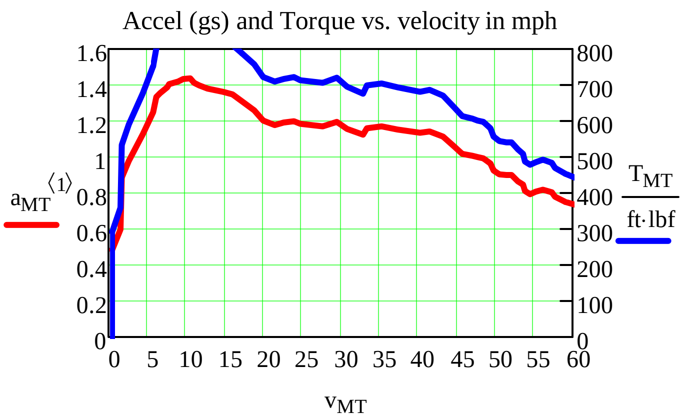


Data points abstracted from Tesla Blue Acceleration Curve



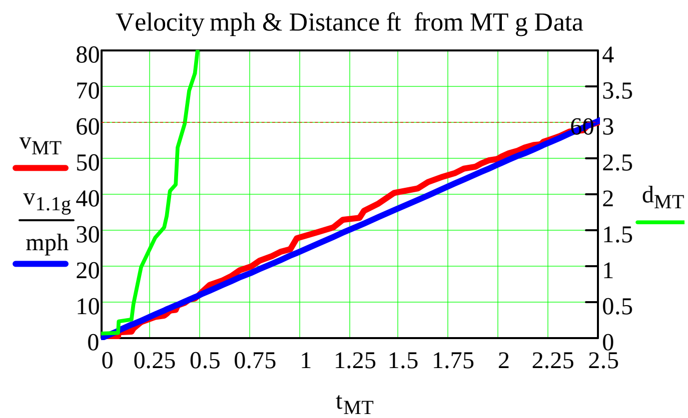
Applied Torque Derived from Acceleration Test Data

$$T_{MT} := M_{gross}\cdot a_{MT}^{\langle 1\rangle}\cdot 22\,\frac{\text{mph}}{\text{s}}\cdot\frac{r_{tire}}{GR}$$



What value of constant g gives 60 mph in 2.5s? It is 1.1g.
Get plot for velocity vs time for g = 1.1g, $v_{1.1g}$

$$\frac{60\text{mph}}{2.5\text{s}} = 1.09\cdot g \qquad v_{1.1g_{uu}} := 1.1g\cdot t_{MT_{uu}}\cdot s$$



V. Model Results and Validation
Meets Ludicrous Plus Specs

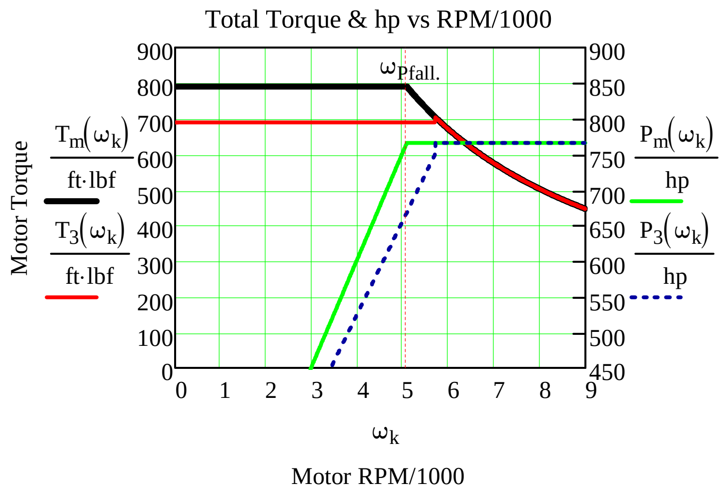
$\text{time}_{a2}(60\cdot\text{mph}) = 2.55\,\text{s}$ $\quad \text{distance}_2(9) = 0.2\cdot\text{mile}$

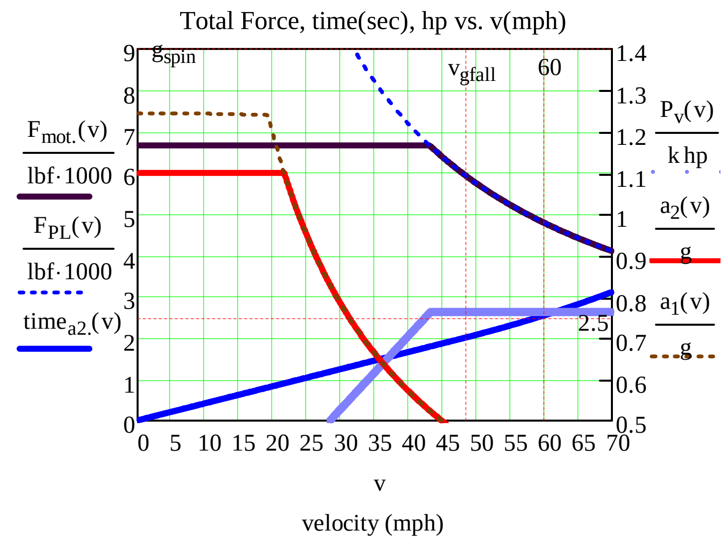
$\text{distance}_2(10.1) = 0.25\cdot\text{mile}$

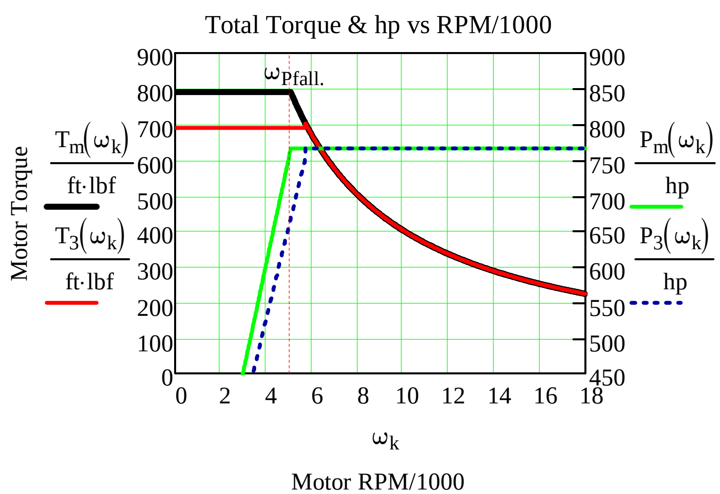
Calculated P100 EPA Range: 297 Miles

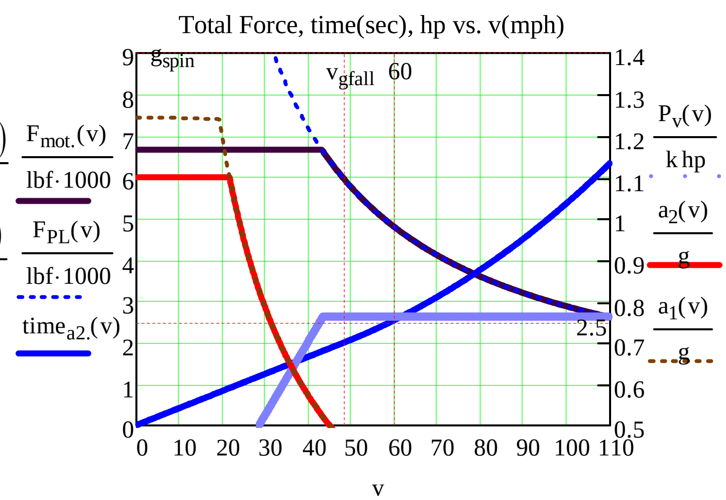
VI. Performance Curve Graphs

Compare with Torque & Power Curves at: http://www.teslamotors.com/performance/acceleration_and_torque.php









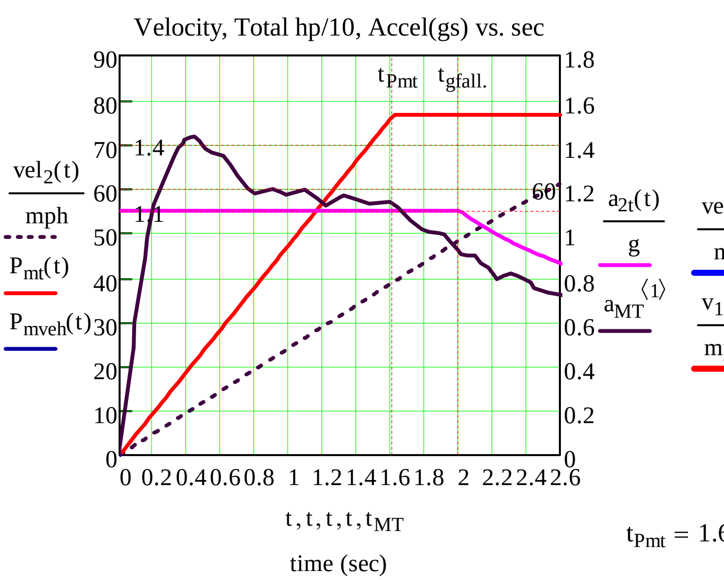
a_{MT} Black curve is Motor Trend (MT) Test acceleration (gs) of S P100D Tesla.

a_{DT} is accel model from Drag Time (DT) P85D Dyno Torque Test.

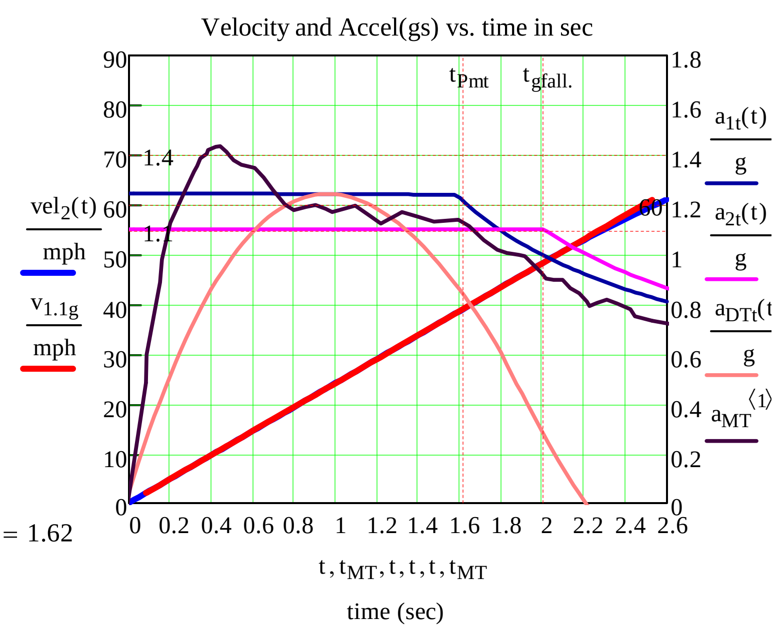
$v_{1.1g}$ shows the plot for constant acceleration of 1.1g

Note: To plot velocity and Power on the same axis, the Power curves were divided by a factor of 10.

g vs.t area for g curves must > area of 1.1g x 2.5s rectangle to reach 60 mph



$t_{Pmt} = 1.62$



VII. Find the Single Charge. Highway Cruise Range for a Given Velocity and Final SOC

Driving Pattern/Profile: Assume we cruise at constant speed, but start, stop, and regen break four times per hour

Drive Train Power Efficiency - Battery Loss for Commanded Vehicle Velocity and Final State of Charge, SOC_f:

SOC_f is 10% at recharge. 400V HV battery idle power is Po. 12V battery gives Accessory Power. The Traction Inverter Efficiency - TInvE, HV Power Electronics at Idle Efficiency - IPEE, and Gear Power Efficiency - GPE are 92.5%, 95%, and 90%, respectively. Brake Regen efficiency of kinetic energy is 64%. Then the number of starts per hour as a function of velocity, NS, NumStarts(v, Po), is

$TInvE := 0.925 \quad IPEE := 0.95 \quad GPE := 0.9 \quad Regen := 0.64$ $\quad$ Change in State of Charge = $1 - SOC_f$

$$Power_{dissLoss}(v, P_o) := \frac{Fo(v)\cdot v}{TInvE\cdot GPE} + \frac{P_o\cdot watt}{IPEE} \qquad Energy_{accel}(v) := Power_{max}\cdot time(v\cdot mph)\cdot hr$$

NSo, NS are iterative converging estimates of total NumStarts per charge

$$NS_o(v) := 2\cdot\left[\frac{65mph}{(v + 0.1\cdot mph)}\right]^2 \qquad NS(v, P_o, SOC_f) := \frac{Energy_{bat}\cdot(1 - SOC_f) - NS_o(v)\cdot\left[\left[\frac{M_{gross}\cdot(v)^2}{2}\cdot(1 - Regen)\right]\right]}{Power_{dissLoss}(v, P_o)\cdot 15\cdot min}$$

$$NumStarts(v, P_o, SOC_f) := floor\left[\frac{Energy_{bat}\cdot(1 - SOC_f) - NS(v, P_o, SOC_f)\cdot\left[\frac{M_{gross}\cdot(v)^2}{2}\cdot(1 - Regen)\right]}{Power_{dissLoss}(v, P_o)\cdot 15\cdot min}\right]$$

$$Cruise\_Range(v, P_o, SOC_f) := \frac{\left[Energy_{bat}\cdot(1 - SOC_f) - NumStarts(v, P_o, SOC_f)\cdot\left[\frac{Regen\cdot M_{gross}\cdot(v)^2}{2}\cdot(1 - Regen)\right]\right]\cdot v}{Power_{dissLoss}(v, P_o)}$$

Highway Cruise Range with Four Stops per Hour Estimate

$Cruise\_Range(30\cdot mph, 100, 0.1) = 358.73\cdot mi$ $\qquad$ $Cruise\_Range(60\cdot mph, 100, 0.1) = 267.7\cdot mi$

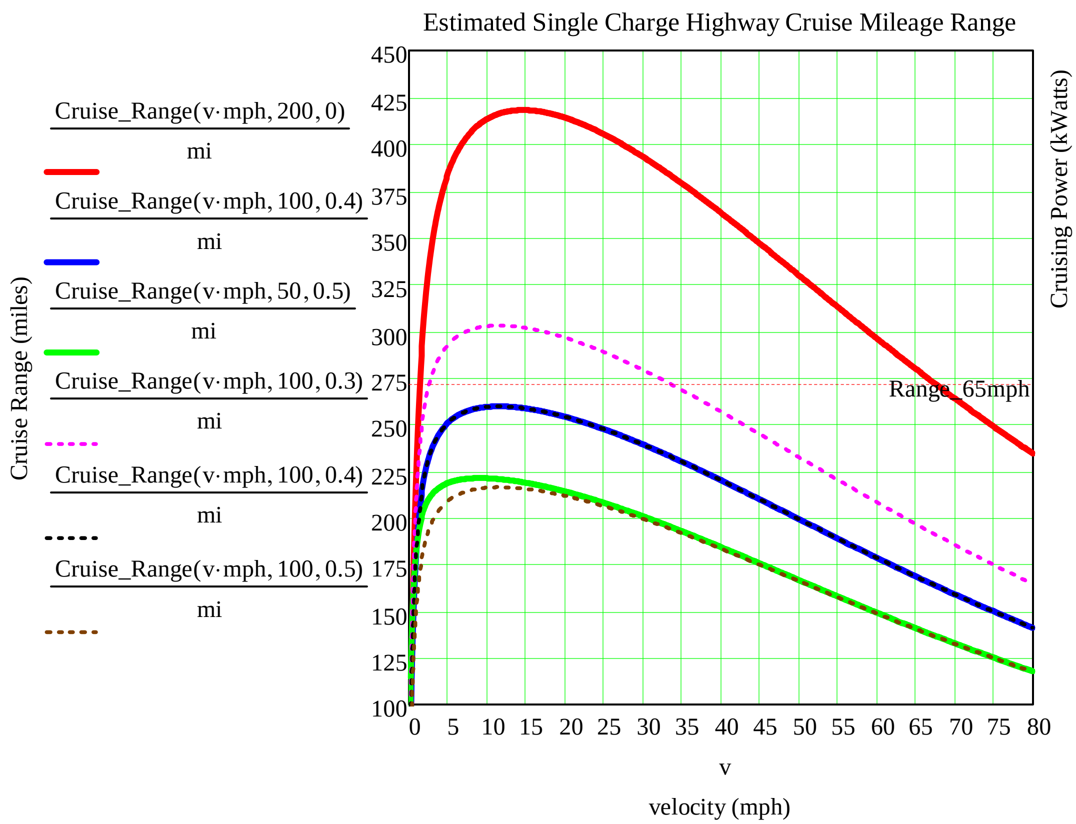
$Cruise\_Range(40\cdot mph, 100, 0.1) = 330.03\cdot mi$ $\qquad$ $Cruise\_Range(70\cdot mph, 100, 0.1) = 238.32\cdot mi$

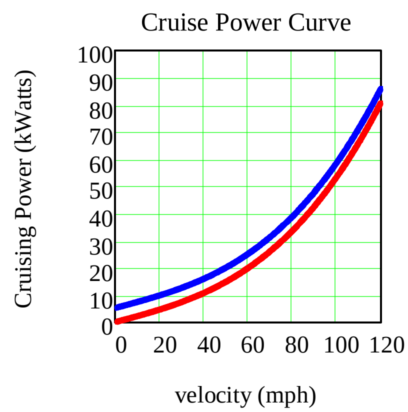
$Cruise\_Range(50\cdot mph, 100, 0.1) = 298.8\cdot mi$ $\qquad$ $Cruise\_Range(60\cdot mph, 200, 0) = 295.87\cdot mi$

Opposing Force-Cruise Power Dissipation, 50 & 5kW

$Power_{cruise}(v, P_o) := Power_{dissLoss}(v, P_o)$

$Power_{cruise}(60\cdot mph, 500) = 2.04\times 10^4\cdot W$





VIII. Find Mileage Range: Use 3 Different EPA Driving Schedules

Algorithm to Calculate Range, Range(P,fHz), 100% Battery Discharge, Driving Profile Velocity/Time File, P and Sampling Rate, fHz

$Energy_{bat} := 100 \cdot kW \cdot hr$

$Range(P, f_{Hz}) :=$

$Ebat \leftarrow E_{diss} \leftarrow v_{old} \leftarrow 0$

$n \leftarrow -1$

$N \leftarrow rows(P) - 1$

while $\left(E_{diss} < \dfrac{Energy_{bat}}{kW \cdot hr} \right)$

$n \leftarrow n + 1$

$t \leftarrow mod(n, N)$

$v \leftarrow P_t$

$$P_{accel} \leftarrow \frac{k_m \cdot M_{gross} \cdot \left(v^2 - v_{old}^2\right) \cdot \frac{mph \cdot f_{Hz}}{sec} mph}{TInvE \cdot GPE \cdot 2} \quad \text{if } v > v_{old}$$

$$P_{accel} \leftarrow k_m \cdot M_{gross} \cdot \left(v^2 - v_{old}^2\right) \cdot \frac{mph \cdot f_{Hz}}{2sec} mph \cdot Regen \quad \text{otherwise}$$

If decelerating, charge battery with Regen fraction of energy.

$$E_{diss} \leftarrow E_{diss} + \frac{\left(Power_{dissLoss}(v \cdot mph, 100) + P_{accel}\right) \cdot sec}{kW \cdot hr \cdot f_{Hz}}$$

$v_{old} \leftarrow v$

$Ebat_n \leftarrow E_{diss}$

$$Range \leftarrow \sum_{m=0}^{n} \frac{\left(P_{mod(m, N)} + P_{mod(m+1, N)}\right) \cdot mph \cdot sec}{2 \cdot mi \cdot f_{Hz}}$$

Read US06 and FTP Dynamometer Drive Profile Files

Refer to: http://www.epa.gov/nvfel/testing/dynamometer.htm

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds. Sampling can be either 1 Hz or 10Hz

The **Federal Test Procedure (FTP)** is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. 10 Hz Sampling data is named FP10 and HY10 for the Highway schedule.

FTPF := READPRN("FedTestProc.txt") $\quad t := FTPF^{\langle 0 \rangle} \quad FTP := FTPF^{\langle 1 \rangle} \quad rows(FTP) = 1875$

UDDSF := READPRN("uddscol.txt") $\quad UDDS := UDDSF^{\langle 1 \rangle} \quad rows(UDDS) = 1370$

HWYF := READPRN("hwycol.txt") $\quad HWY := HWYF^{\langle 1 \rangle} \quad R_{hwy} := rows(HWY)$

FP10 := READPRN("FTP10Hz.TXT") $\quad FTP10V := submatrix(FP10, 0, rows(FP10) - 1, 1, cols(FP10) - 1)$

HY10 := READPRN("HWY10Hz.TXT") $\quad HWY10V := submatrix(HY10, 0, rows(HY10) - 1, 1, cols(HY10) - 1)$

US06F := READPRN("US06PROFILE.TXT") $\quad Time := US06F^{\langle 0 \rangle} \quad US06 := US06F^{\langle 1 \rangle} \quad n_6 := 0 .. 598$

$r1 := 0 .. rows(HY10) \cdot 10 - 1 \quad HWY10_{r1} := HWY10V_{ceil\left(\frac{r1+1}{10}\right)-1, mod(r1, 10)}$

Using EPA Profiles and above Range Program, Calculate Tesla EV Range for EPA Profiles

$Range_{US06} := Range(US06, 1)$ $Range_{FTP} := Range(FTP, 1)$ $Range_{HWY} := Range(HWY, 1)$

EPA 2008 Cycle MPG Fuel Economy Least Squares Fit Regression for Range

$$MPG_{city} := \frac{1}{\left(0.003259 + \frac{1.18053}{Range_{FTP}}\right)} \qquad MPG_{hwy} := \frac{1}{0.001376 + \frac{1.3466}{Range_{HWY}}}$$

$$MPG_{epa} := 0.55 \cdot MPG_{city} + 0.45 \cdot MPG_{hwy}$$

Single Charge EPA Range Calculations: Federal Test Procedure (FTP), Highway, and US06

Model Validation: **Published EPA Range is 315 miles**

$Range_{FTP} = 272.46$ $Range_{HWY} = 290.79$ $Range_{US06} = 213.33$

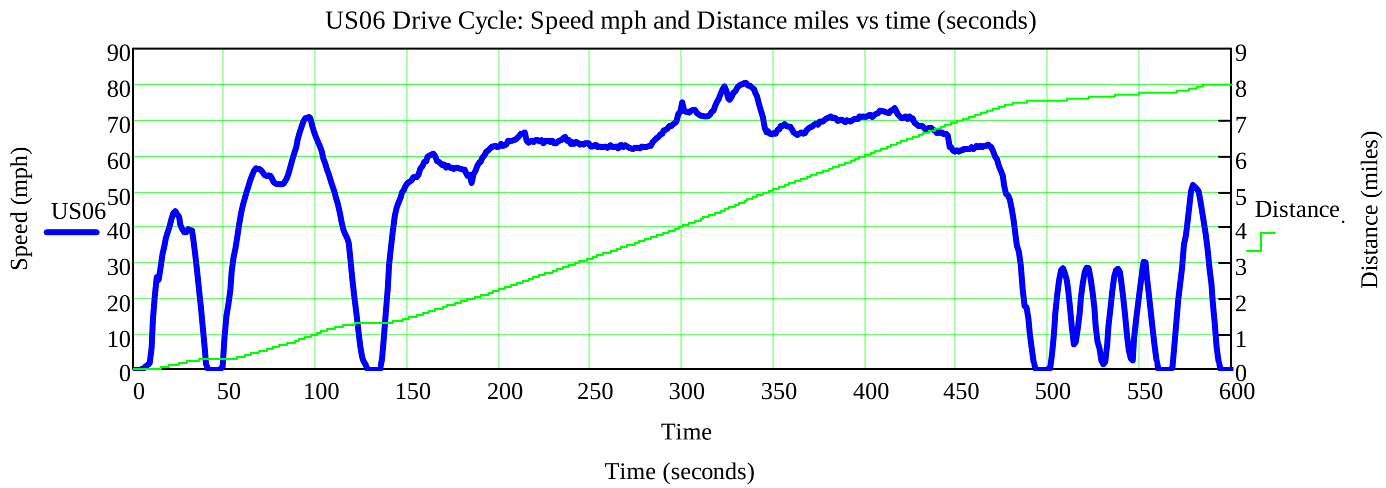
$MPG_{city} = 131.72$ $MPG_{hwy} = 166.48$ $MPG_{epa} = 147.36$

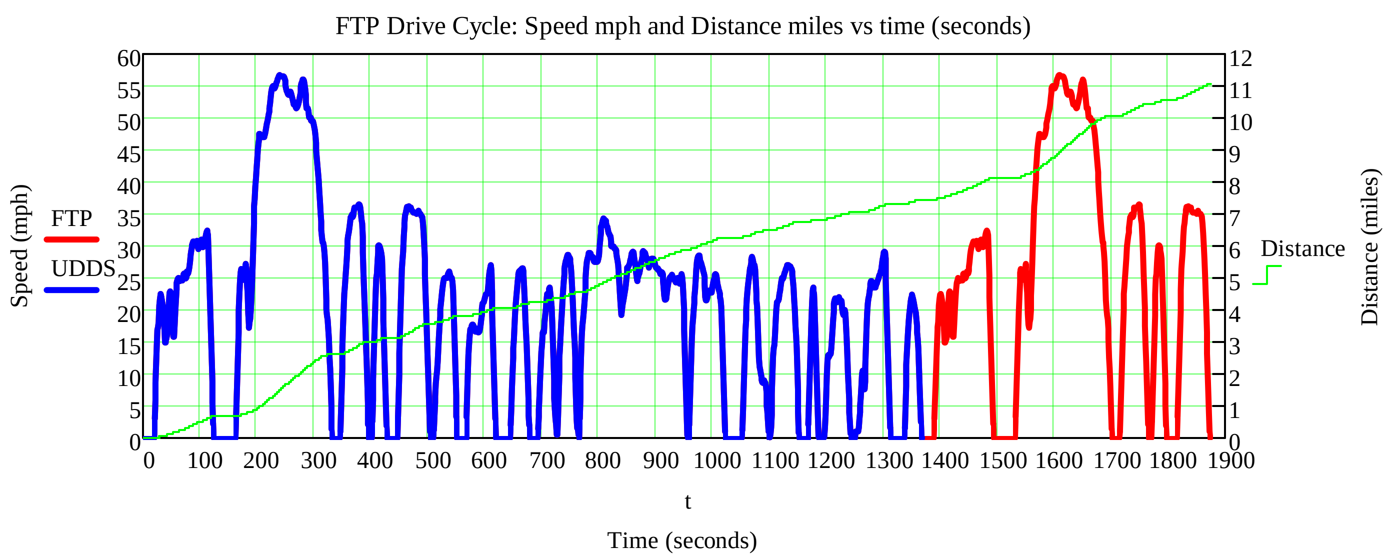
$r := 0 .. rows(FTP) - 1$ $Distance_r := \sum_{r=0}^{r} FTP_r \cdot \frac{1}{60 \cdot 60}$ $rr := 0 .. rows(US06) - 1$ $Distance_{\cdot rr} := \sum_{rr=0}^{rr} US06_{rr} \cdot \frac{1}{60 \cdot 60}$

$max(Distance) = 11.04$

$max(Distance) = 11.04$ $max(Distance_{\cdot}) = 8.01$

Plots of EPA Dynamometer Vehicle Testing Profiles





IX. Drag Times Tesla P85D Torque and Power Dynamometer

http://www.dragtimes.com/2015-Tesla-Model-S-Videos-27143.html

Extract Points from Curves to csv data files

Read Dyno Data csv data files

$T_{Dyn} := \text{READPRN}(\text{"Tesla P85D DynoTorq Insane.csv"})$ $\quad T_{Dyn}^{\langle 0 \rangle} := T_{Dyn}^{\langle 0 \rangle} - 5.916$ $\quad \text{rows}(T_{Dyn}) = 27$

$P_{Dyn} := \text{READPRN}(\text{"Tesla P85D DynPowerR.csv"})$ $\quad P_{Dyn}^{\langle 0 \rangle} := P_{Dyn}^{\langle 0 \rangle} - 5.916$ $\quad \text{rows}(P_{Dyn}) = 24$

Power Series Fit to Torque to get Shape Curve

Guess $\quad a := 1 \quad b := 1 \quad c := 1 \quad d := 27 \quad e := 1 \quad f := 30 \quad gx := 0$

Given $\quad Torq := T_{Dyn}^{\langle 1 \rangle} \quad spd := T_{Dyn}^{\langle 0 \rangle}$

$Torq - \left[a\cdot(spd)^3 - c\cdot(spd - d)^2 - e\cdot(spd - f) + gx\right] = 0$ $\qquad At := \text{Minerr}(a, c, d, e, f, gx)$

$\text{Torque}(s) := At_0\cdot(s)^3 - At_1\cdot(s - At_2)^2 - At_3\cdot(s - At_4) + At_5$ $\qquad \text{Torque}(0) = 23.24$

Guess $\quad a := 1 \quad b := 1 \quad c := 1 \quad d := 1 \quad e := 1 \quad f := 30 \quad gx := 0 \quad h := 0 \quad i := 0$

Given $\quad P_{wheel} := P_{Dyn}^{\langle 1 \rangle} \quad spd := P_{Dyn}^{\langle 0 \rangle} \quad n := 0..159 \quad S_n := n\cdot 0.5 \quad Tx_n := \text{Torque}(S_n)$

$P_{wheel} - \left[a\cdot(spd - b)^3 - c\cdot(spd - d)^2 - e\cdot(spd - f) + gx\cdot(spd - h) + i\right] = 0$ $\qquad A1 := \text{Minerr}(a, b, c, d, e, f, gx, h, i)$

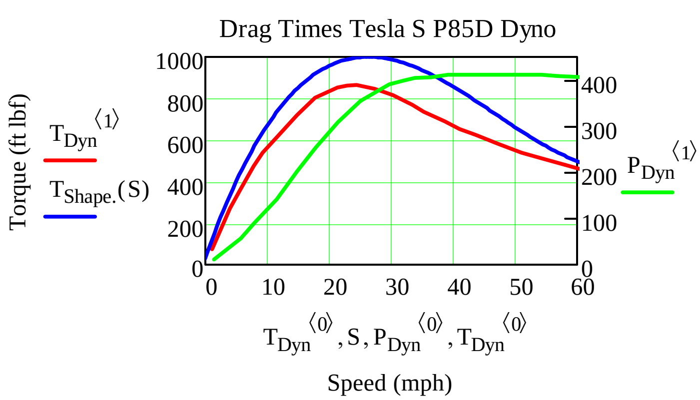
$\text{Pwr}(s) := A1_0\cdot(s - A1_1)^0 - A1_2\cdot(s - A1_3)^2 - A1_4\cdot(s - A1_5) + A1_6\cdot(s - A1_7) + A1_8$ $\qquad \text{Pwr}(30) = 344.2$

$P_{Dyn\_Fv} := \overrightarrow{\left(T_{Dyn}^{\langle 1 \rangle}\dfrac{ft\cdot lbf}{r_{tirex}\cdot in}\cdot T_{Dyn}^{\langle 0 \rangle}\cdot\dfrac{mph}{14\cdot hp}\right)}$ $\qquad P_{Dyn\_Fv_{20}} = 177.41\,m^{-1}$

Normalize Torque Curve to Max = 1 to extract shape only, TShape

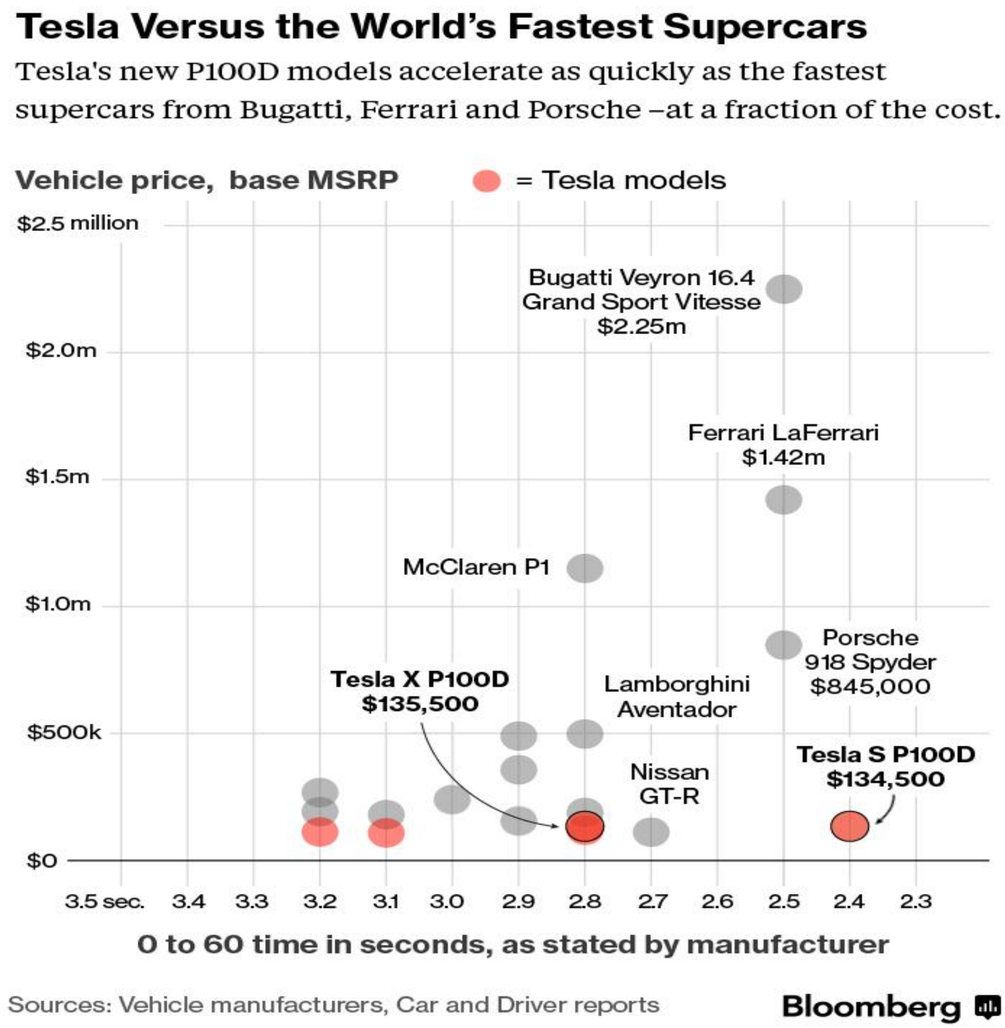
Torque Shape: $\quad NormT := \dfrac{1}{\max(Tx)} \quad T_{Shape}(s) := \text{Torque}(s)\cdot NormT \quad T_{Shape.}(s) := T_{Shape}(s)\cdot 1000$

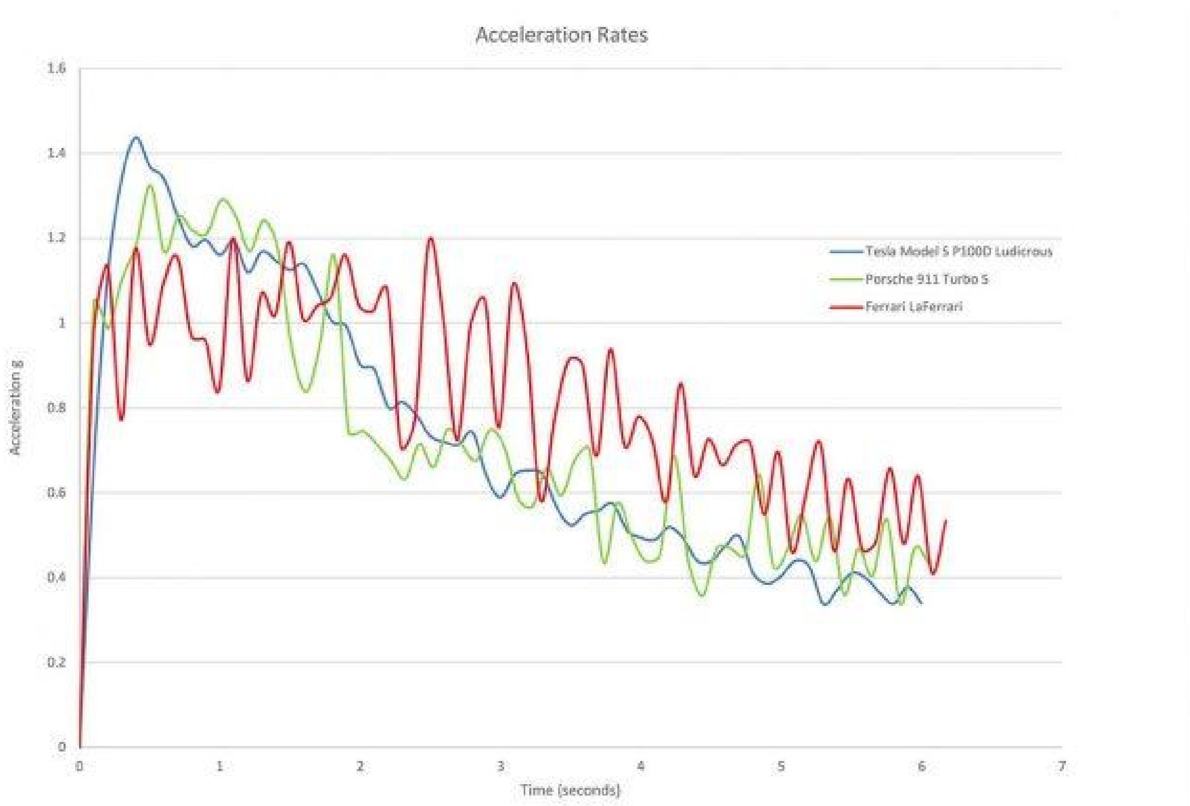
Multiple by 1000 to plot on same axis with Original Torque Curve



$$At = \begin{pmatrix} 0.01 \\ 1.88 \\ 7.86 \\ -41.57 \\ -154.22 \\ -6270.96 \end{pmatrix} \qquad A1 = \begin{pmatrix} 0 \\ 6.65 \\ 0.36 \\ -21.66 \\ 9.2 \\ 278.18 \\ 49.07 \\ 18.22 \\ -1557.66 \end{pmatrix}$$

X.





Source: http://www.motortrend.com/cars/tesla/model-s/2017/2017-tesla-model-s-p100d-first-test-review/

XII. Tire Friction (Composition and Width)

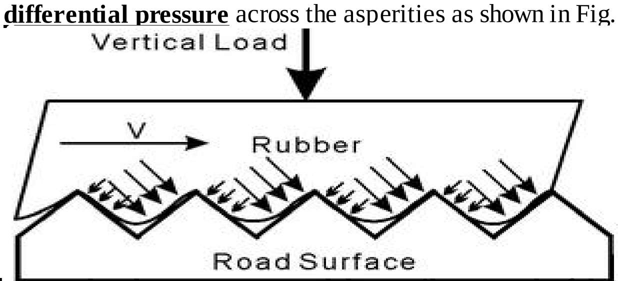
Coefficient of Static Friction (μ) is the ratio of Tire Road Force to Vehicle Weight. Values of μ for Conventional Car tire On: Asphalt 0.72, Car tire Grass 0.35.

Top Fuel drag car tires are getting a coefficient of friction well **over 4.5. How is this possible?**

This material came from: http://insideracingtechnology.com/tirebkexerpt1.htm **See Mathcad/EVs/Tire Friction.doc**

Rubber generates friction in three major ways: **adhesion, deformation, and wear.**

Rubber in contact with a **smooth surface** (glass is often used in testing) generates friction forces mainly by **adhesion**. **When rubber is in contact with a rough surface, another mechanism, deformation, comes into play. Movement** of a rubber slider on a r**ough surface** results in the **deformation of the rubber by high points o**n the surface called irregularities or **asperities**. A load on the rubber slider causes the asperities to **penetrate the rubber** and the **rubber drapes ove**r the asperities. The **energy needed** to move the asperities in the rubber comes from the **differential pressure** across the asperities as shown in Fig. 3.4, where a rubber slider moves on an irregular surface at speed V.



Tearing and Wear

As deformation forces and sliding speeds go up, **local stress** can e**xceed the tensile strength of the rubber**, especially at an increase in local stress near the point of a **sharp irregularity**. High local stress can **deform the internal structure of the rubber past the point of elastic recovery.** When polymer bonds and crosslinks are **stressed to failure the material can't recover completely**, and this can cause **tearing. Tearing absorbs energy**, resulting in **additional friction forces** in the contact surface.

Wear is the ultimate result of tearing.

$$F_{total} = F_{adhesive} + F_{defformation} + F_{wear}$$

Deformation Friction and Viscoelasticity

Rubber is elastic and conforms to surface irregularities. But rubber is **also viscoelastic**; it **doesn't rebound fully** after deformation.

Hysteresis

Hysteresis, or energy loss, in rubber.
where there is **some sliding** between the rubber and an irregular surface. If the **rubber recovers slowly** from the passing irregularity as in the high-hysteresis rubber, it **can't push o**n the downstream surfaces of the irregularities **as hard** as it pushes on the upstream surfaces. This **pressure differenc**e between the **upstream and downstream faces of the irregularity** results in **friction forces** even when the surfaces are lubricated.

Wide Tires: It is true that wider tires commonly have better traction. The main reason why this is so does not relate to contact patch, however, but to **composition. Soft compound tires** are required to be **wider in order for the side-wall to support the weight** of the car. softer tires have a larger coefficient of friction, therefore better traction. A narrow, soft tire would not be strong enough, nor would it last very long. **Wear in a tire is related to contact patch. Harder compound tires wear much longer**, and can be narrower. They do, however have a lower coefficient of friction, therefore less traction. Among tires of the same type and composition, here is no appreciable difference in 'traction' with different widths. **Wider tires**, assuming all other factors are equal, commonly have **stiffer side-walls and experience less roll. This gives better cornering performance.**

Friction is proportional to the normal force of the asphalt acting upon the car tires. This force is simply equal to the weight which is distributed to each tire when the car is on level ground. Force can be stated as Pressure X Area. **For a wide tire, the area is large but the force per unit area is small** and vice versa. The force of friction is therefore the same whether the tire is wide or not. However, **asphalt is not a uniform surface**. Even with steamrollers to flatten the asphalt, the surface is still somewhat irregular, especially over the with of a tire. Drag racers can therefore **increase the probability or likelihood of making contact** with the road by using a wider tire. In addition a secondary benefit is that the wider tire increased the support base a

F**riction force is independent of the apparent area of contact.** **For hard materials**, this is nearly correct. The true area of contact varies with the applied load. The apparent area does not. If you can imagine the contact zone from a **microscopic viewpoint, only a tiny portion of the apparent area actually touches.** This tiny area is the true area of contact. But this applies to hard materials. It does **not apply to elastomers, such as rubber.** Tire tread rubber compounds vary greatly from one application to another. **Race car tire** tread compounds can be **very soft, viscoelastic materials**, while heavy truck tread rubber can be quite hard. In general, **soft rubber materials have greater friction**. With drag racing 'slicks,' the tire tread material **literally sticks** to the pavement--and the r**ubber is sheared from the tire.** Clearly, the greater the apparent contact area, the greater this shear force. **Cleanliness is important** to getting the surfaces to '**stick.**' This is one reason why drag racers have a '**burn-out'** before each race (another is to raise the tire tread surface temperature). However, there is another reason for wide tire treads on some road and track racing cars. They need **tread volume to provide enough wear life**. Tires wear rapidly under racing conditions. **Some long races wear out several sets of tires.** There are **trade-offs with traction and tread life.** That is why heavy truck tire tread compounds do not have as much friction as those used on passenger cars. However, truck tire tread compounds provide longer wear life and less heat build-up. Like many things in this world, **tire tread choices involve compromises.**